POWER SERIES QUIZ

BLAKE FARMAN

Lafayette College

Name: Solutions

1. Determine the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(x-1)^n$$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-1)^{n+1}}{(2(n+1)-1)2^{n+1}}\frac{(2n-1)2^n}{(-1)^n(x-1)^n}\right| = \lim_{n\to\infty}\frac{|x-1|^{n+1}}{|x-1|^n}\frac{2^n}{2^{n+1}}\frac{(2n-1)}{(2n+1)}$$

$$= \lim_{n\to\infty}\frac{|x-1|}{2}\frac{(2n-1)}{(2n+1)} = \frac{|x-1|}{2} < 1 \Rightarrow |x-1| < 2 \Leftrightarrow -2 < x-1 < 2$$
$$\Leftrightarrow -1 < x < 3.$$

$\underline{x=-1}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n(-1-1)^n}{(2n-1)2^n} = \sum_{n=1}^{\infty}\frac{(-1)^n(-2)^n}{(2n-1)2^n} = \sum_{n=1}^{\infty}\frac{2^n}{(2n-1)2^n} = \sum_{n=1}^{\infty}\frac{1}{2n-1}$$

This series diverges by Limit Comparison with the Harmonic Series:

$$\lim_{n\to\infty} \frac{1}{2n-1}\Big/\frac{1}{n} = \lim_{n\to\infty}\frac{n}{2n-1} = \frac{1}{2} > 0$$

$\underline{x=3}$

$$\sum_{n=1}^{\infty}\frac{(-1)^n(3-1)^n}{(2n-1)2^n} = \sum_{n=1}^{\infty}\frac{(-1)^n 2^n}{(2n-1)2^n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}$$ converges by the A.S.T. since

$2n-1$ is increasing so $b_n = \left|\frac{(-1)^n}{2n-1}\right| = \frac{1}{2n-1}$ is decreasing and $\lim_{n\to\infty} b_n = 0$.