GEOMETRIC SERIES

BLAKE FARMAN

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Name: ____

Theorem. *The geometric series*

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \dots$$

converges if $|r| < 1$ *and diverges otherwise. The sum of the convergent series is*

$$s = \frac{a}{1-r},\ |r| < 1.$$

Observation: For a geometric series, we can always find the value of r by taking the ratio of any two consecutive terms:

$$\frac{a_{n+1}}{a_n} = \frac{ar^n}{ar^{n-1}} = r.$$

1. Determine whether the series

$$4 + 3 + \frac{9}{4} + \frac{27}{16} + \dots$$

converges or diverges. If it is convergent, find its sum.

2. Determine whether the series

$$\sum_{k=1}^{\infty} \frac{k(k+14)}{(k+15)^2}$$

converges or diverges. If it converges, find its sum.

3. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2^n+1}{3^n}$$

converges or diverges. If it converges, find its sum.

4. Express $0.\bar{8}$ as a rational number (i.e. a ratio of two integers).

5. Express $2.\overline{516}$ as a rational number.