DIFFERENTIAL EQUATIONS

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Name: Solutions

Find the solution of the differential equation that satisfies the given initial condition.

1. $y' = xe^y$, $y(0) = 0$.

$\int e^{-y} dy = \int x dx \Rightarrow -e^{-y} = \frac{1}{2}x^2 + C$

$\Rightarrow e^{-y} = -\frac{1}{2}x^2 + C$

$\Rightarrow -y = \ln(-\frac{1}{2}x^2 + C)$

$\Rightarrow y = -\ln(-\frac{1}{2}x^2 + C)$

$0 = -\ln(-\frac{1}{2}0^2 + C)$

$= -\ln(C)$

$\Rightarrow C = 1$

$\boxed{y = -\ln(-\frac{1}{2}x^2 + 1)}$

2. $y' = \dfrac{x\sin(x)}{y}$, $y(0) = -1$.

$u = x \quad v = -\cos(x)$

$du = dx \quad dv = \sin(x)dx$

$\int y\, dy = \int x\sin(x)dx$

$\Rightarrow \frac{1}{2}y^2 = -x\cos(x) + \int \cos(x)dx$

$= -x\cos(x) + \sin(x) + C$

$\Rightarrow y = -\sqrt{2x\cos(x) + 2\sin(x) + C}$

$-1 = -\sqrt{2(0)\cos(0) + 2\sin(0) + C}$

$= -\sqrt{C}$

$\Rightarrow C = 1.$

$\boxed{y = -\sqrt{2x\cos(x) + 2\sin(x) + 1}}$

3. $P' = \sqrt{Pt}$, $P(1) = 2$.

$$\int \frac{dP}{\sqrt{P}} = \int \sqrt{t}\, dt \Rightarrow 2\sqrt{P} = \frac{2}{3} t^{3/2} + C$$

$$\Rightarrow P = \left(\frac{1}{3} t^{3/2} + C\right)^2$$

$$2 = \left(\frac{1}{3} + C\right)^2 \Rightarrow \sqrt{2} = \frac{1}{3} + C$$

$$\Rightarrow C = -\frac{1}{3} + \sqrt{2}$$

$$\boxed{P = \left(\frac{1}{3} t^{3/2} - 1/3 + \sqrt{2}\right)^2}$$

4. $y' = -x/y$, $y(0) = 3$.

$$\int y\, dy = \int -x\, dx$$

$$\Rightarrow \frac{1}{2} y^2 = -\frac{1}{2} x^2 + C$$

$$\Rightarrow y^2 + x^2 = C$$

$$C = 3^2 + 0^2 = 9$$

$$\boxed{y = \sqrt{9 - x^2}}$$