LIMITS AT INFINITY

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Name: Solutions

Evaluate the following limits at infinity.

1. $\lim\limits_{x\to\infty} \dfrac{3x-2}{2x+1} = \lim\limits_{x\to\infty} \dfrac{x}{x}\left(\dfrac{3-2/x}{2+1/x}\right)$

$= \lim\limits_{x\to\infty} \dfrac{3-2/x}{2+1/x}$

$= \dfrac{3-0}{2+1} = \boxed{\dfrac{3}{2}}$

2. $\lim\limits_{x\to\infty} \dfrac{4x^3+6x^2-2}{2x^3-4x+5} = \lim\limits_{x\to\infty} \dfrac{x^3(4+6/x-2/x^3)}{x^3(2-4/x^2+5/x^3)}$

$= \lim\limits_{x\to\infty} \dfrac{4+6/x-2/x^3}{2-4/x^2+5/x^3}$

$= \dfrac{4+0-0}{2-0+0} = \dfrac{4}{2} = \boxed{2}$

3. $\lim\limits_{x\to\infty} \dfrac{\sqrt{2x^2+1}}{3x-5} = \lim\limits_{x\to\infty} \dfrac{\sqrt{x^2(2+1/x)}}{x(3-5/x)}$

$= \lim\limits_{x\to\infty} \dfrac{x\sqrt{2+\frac{1}{x}}}{x(3-5/x)} = \lim\limits_{x\to\infty} \dfrac{\sqrt{2+\frac{1}{x}}}{(3-5/x)} = \dfrac{\sqrt{2+0}}{3-0} = \boxed{\dfrac{\sqrt{2}}{3}}$

4. $\lim_{x\to\infty} \dfrac{x+3x^2}{4x-1} = \lim_{x\to\infty} \dfrac{x^2}{x}\left(\dfrac{\frac{1}{x}+3}{4-\frac{1}{x}}\right)$

$= \lim_{x\to\infty} x\left(\dfrac{\frac{1}{x}+3}{4-\frac{1}{x}}\right) = \boxed{\infty}$

5. $\lim_{x\to\infty} \dfrac{x^3-x}{x^2-6x+5} = \lim_{x\to\infty} \dfrac{x^3(1-\frac{1}{x^2})}{x^2(1-\frac{6}{x}+\frac{5}{x^2})}$

$= \lim_{x\to\infty} x\left(\dfrac{1-\frac{1}{x^2}}{1-\frac{6}{x}+\frac{5}{x^2}}\right)$

$= \boxed{\infty}$

6. $\lim_{x\to\infty} \dfrac{x^4-3x^2+x}{x^3-x+2} = \lim_{x\to\infty} \dfrac{x^4(1-\frac{3}{x^2}+\frac{1}{x^3})}{x^3(1-\frac{1}{x^2}+\frac{2}{x^3})}$

$= \lim_{x\to\infty} x\left(\dfrac{1-\frac{3}{x^2}+\frac{1}{x^3}}{1-\frac{1}{x^2}+\frac{2}{x^3}}\right)$

$= \boxed{\infty}$

7. $\lim\limits_{x\to\infty} \dfrac{1-x^2}{x^3-x+1} = \lim\limits_{x\to\infty} \dfrac{x^2}{x^3}\left(\dfrac{1/x^2-1}{1-1/x^2+1/x^3}\right)$

$$= \lim_{x\to\infty} \frac{1}{x}\left(\frac{1/x^2-1}{1-1/x^2+1/x^3}\right)$$

$$= 0\left(\frac{0-1}{1-0+0}\right) = \boxed{0}$$

8. $\lim\limits_{x\to\infty} \dfrac{1+x^4}{x^6+1} = \lim\limits_{x\to\infty} \dfrac{x^4(1/x^4+1)}{x^6(1+1/x^6)}$

$$= \lim_{x\to\infty} \frac{1}{x^2}\left(\frac{1/x^4+1}{1+1/x^6}\right)$$

$$= 0\left(\frac{0+1}{1+0}\right) = \boxed{0}$$

9. $\lim\limits_{x\to\infty} \dfrac{x-2}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{x}{x^2}\left(\dfrac{1-2/x}{1+1/x^2}\right)$

$$= \lim_{x\to\infty} \frac{1}{x}\left(\frac{1-2/x}{1+1/x^2}\right)$$

$$= 0\left(\frac{1-0}{1+0}\right) = \boxed{0}$$

10. $\lim_{x\to\infty} \left(\sqrt{9x^2+x} - 3x\right) = \lim_{x\to\infty} \left(\sqrt{9x^2+x} - 3x\right)\left(\frac{\sqrt{9x^2+x} + 3x}{\sqrt{9x^2+x} + 3x}\right)$

$$= \lim_{x\to\infty} \frac{9x^2+x-\cancel{9x^2}}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2(9+1/x)}+3x}$$

$$= \lim_{x\to\infty} \frac{x}{\cancel{x}\left(\sqrt{9+1/x}+3\right)}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\cancel{1/x}}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

11. $\lim_{x\to\infty} \left(x-\sqrt{x}\right) = \lim_{x\to\infty} x\left(1-\frac{1}{\sqrt{x}}\right) = \boxed{\infty}$