DERIVATIVES

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Name: Solutions

In each of the following problems, use the **limit definition**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

to compute the derivative of the given function.

1. $g(x) = \dfrac{x+3}{x+5}$

$$g'(x) = \lim_{h\to 0} \frac{\frac{(x+h)+3}{(x+h)+5} - \frac{(x+3)}{(x+5)}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{(x+h+3)(x+5) - (x+3)(x+h+5)}{(x+h+5)(x+5)}\right)$$

$$= \lim_{h\to 0} \left(\frac{\cancel{x^2} + \cancel{5x} + \cancel{hx} + 5h + \cancel{3x} + \cancel{15} - (\cancel{x^2} + \cancel{hx} + \cancel{5x} + \cancel{3x} + 3h + \cancel{15})}{h(x+h+5)(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{5h - 3h}{h(x+h+5)(x+5)}$$

$$= \lim_{h\to 0} \frac{2h}{h(x+h+5)(x+5)}$$

$$= \lim_{h\to 0} \frac{2}{(x+h+5)(x+5)} = \boxed{\frac{2}{(x+5)^2}}$$

2. $h(x) = \sqrt{9-x}$

$$\lim_{h\to 0} \frac{\sqrt{9-(x+h)} - \sqrt{9-x}}{h} = \lim_{h\to 0} \frac{\sqrt{9-x-h} - \sqrt{9-x}}{h} \frac{\left(\sqrt{9-x-h} + \sqrt{9-x}\right)}{\left(\sqrt{9-x-h} + \sqrt{9-x}\right)}$$

$$= \lim_{h\to 0} \frac{(9-x-h) - (9-x)}{h\left(\sqrt{9-x-h} + \sqrt{9-x}\right)} = \lim_{h\to 0} \frac{\cancel{9}\cancel{-x}-h - \cancel{9}+\cancel{x}}{h\left(\sqrt{9-x-h} + \sqrt{9-x}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{h\left(\sqrt{9-x-h} + \sqrt{9-x}\right)} = \lim_{h\to 0} \frac{-1}{\left(\sqrt{9-x-h} + \sqrt{9-x}\right)}$$

$$= \frac{-1}{\sqrt{9-x-0} + \sqrt{9-x}} = \boxed{\frac{-1}{2\sqrt{9-x}}}$$