CONTINUITY

BLAKE FARMAN

Lafayette College

Name: _____

Definition. A function, f, is **continuous** at a if

$$\lim_{x \to a} f(x) = f(a)$$

1. Use the definition to show that the given function is continuous at the given number, a.

(a) $f(t) = \dfrac{t^2 + 5t}{2t + 1}$, $a = 2$.

(b) $f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}$, $a = 2$.

2. Show that the function

$$f(x) = \frac{x-1}{3x+6}$$

is continuous on the interval $(-\infty, -2) \cup (-2, \infty)$.

3. Find the number k that makes the function

$$f(x) = \begin{cases} \dfrac{x^3 - 8}{x^2 - 4} & x \neq 2 \\[2mm] k & x = 2 \end{cases}$$

continuous.

4. Use continuity to evaluate the given limit.

(a) $\lim\limits_{x \to \pi} \sin\left(x + \sin(x)\right)$

(b) $\lim\limits_{x \to 4} x\sqrt{20 - x^2}$

5. Use the Intermediate Value Theorem to show that there is a solution to the given equation in the specified interval.

Note: You do not need to find the solution!

(a) $x^4 + x - 3 = 0$, $(1, 2)$

(b) $\dfrac{2}{x} = x - \sqrt{x}$, $(2, 3)$

(c) $\cos(x) = x$, $(0, 1)$