FUNDAMENTAL THEOREM OF CALCULUS

BLAKE FARMAN

Lafayette College

Name: Solutions

Use the following theorem to evaluate the given definite integral.

Fundamental Theorem of Calculus, Part II. If $F'(x) = f(x)$ on the interval (a, b), then

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

1. $\displaystyle\int_1^3 (x^2 + 2x - 4)\,\mathrm{d}x$

$$= \int_1^3 x^2 dx + 2\int_1^3 x dx - 4\int_1^3 dx$$

$$= \frac{1}{3}x^3\Big|_1^3 + 2\left(\frac{1}{2}\right)x^2\Big|_1^3 - 4x\Big|_1^3$$

$$= \frac{1}{3}(3^3 - 1^3) + (3^2 - 1^2) - 4(3-1)$$

$$= \frac{1}{3}(26) + (8) - 8$$

$$= \boxed{\frac{26}{3}}$$

2. $\displaystyle\int_0^1 (1 - 8v^3 + 16v^7)\,\mathrm{d}v$

$$= \int_0^1 dv - 8\int_0^1 v^3 dv + 16\int_0^1 v^7 dv$$

$$= v\Big|_0^1 - 8\left(\frac{1}{4}\right)v^4\Big|_0^1 + 16\left(\frac{1}{8}\right)v^8\Big|_0^1$$

$$= (1-0) - 2(1^4 - 0^4) + 2(1^8 - 0^8)$$

$$= 1 - 2 + 2$$

$$= \boxed{1}$$

3. $\int_1^8 x^{-2/3}\,\mathrm{d}x$

$$= \frac{x^{1/3}}{\left(\frac{1}{3}\right)}\Big|_1^8$$

$$= 3\left(8^{1/3} - 1^{1/3}\right)$$

$$= 3(2-1)$$

$$= \boxed{3}$$

4. $\int_{\pi/6}^{\pi/2} \csc(t)\cot(t)\,\mathrm{d}t$

$$= -\csc(t)\Big|_{\pi/6}^{\pi/2}$$

$$= -\left(\csc(\pi/2) - \csc(\pi/6)\right)$$

$$= \csc\left(\frac{\pi}{6}\right) - \csc\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{\frac{1}{2}} - \frac{1}{1}$$

$$= 2 - 1$$

$$= \boxed{1}$$

5. $\displaystyle\int_{\pi/4}^{\pi/3} \csc^2(\theta)\,\mathrm{d}\theta = -\cot(\theta)\Big|_{\pi/4}^{\pi/3}$

$$= -\left(\cot\left(\tfrac{\pi}{3}\right) - \cot\left(\tfrac{\pi}{4}\right)\right)$$

$$= \cot\left(\tfrac{\pi}{4}\right) - \cot\left(\tfrac{\pi}{3}\right)$$

$$= 1 - \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= 1 - \frac{1}{2}\,\frac{2}{\sqrt{3}}$$

$$= \boxed{1 - \frac{1}{\sqrt{3}}}$$

6. $\displaystyle\int_{0}^{\pi/4} \sec(\theta)\tan(\theta)\,\mathrm{d}\theta = \sec(\theta)\Big|_{0}^{\pi/4}$

$$= \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} - \frac{1}{1}$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \boxed{\sqrt{2} - 1}$$