MATH 170
EXAM 02

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> Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
> Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**

Name: Solutions - Version 1

Problem	Points Earned	Points Possible
1		20
2		20
3		20
4		20
5		20
Total		100

Date: April 18, 2016.

1. Problems

1 (20 Points). *Consider the system of equations*

$$\begin{aligned} 2x + 3y &= 1 \\ x + 2y &= 2. \end{aligned}$$

(a) Write down the **matrix equation** *(not the augmented matrix) associated to this system.*

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(b) What is the inverse of the coefficient matrix?

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{2\cdot 2 - 3\cdot 1}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{4-3}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

(c) Use your answers from the previous parts to solve the system.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\cdot 1 + (-3)(2) \\ (-1)(1) + 2\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 6 \\ -1 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4 \\ 3 \end{bmatrix}.$$

2 (20 Points). *Solve the system*

$$\begin{aligned} 2x - 3y &= 3 \\ -x + 2y &= 4 \end{aligned}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \cdot 3 + 3 \cdot 4 \\ 1 \cdot 3 + 2 \cdot 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 12 \\ 3 + 8 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 11 \end{bmatrix}.$$

3 (20 Points). *Solve the system of equations*

$$\begin{aligned} x + 3y + z &= 4 \\ 3x + 3z &= 12. \end{aligned}$$

If there is no solution, simply write 'no solution.' If the system is dependent, express your answer in terms of x, where $y = y(x)$ and $z = z(x)$.

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 3 & 0 & 3 & 12 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 1 & 0 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 0 & -3 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & -3 & 0 & 0 \end{array}\right] \Leftrightarrow \begin{aligned} x + z &= 4 \\ -3y &= 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} z &= 4 - x \\ y &= 0 \end{aligned}$$

so the solution is

$(x, 0, 4-x)$.

4 (20 Points). *Solve the system of equations*

$$\begin{aligned} x + z &= 4 \\ 2y &= 4 \\ 3x - 3z &= -6 \end{aligned}$$

If there is no solution, simply write 'no solution.' If the system is dependent, express your answer in terms of x, where $y = y(x)$ and $z = z(x)$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 2 & 0 & 4 \\ 3 & 0 & -3 & -6 \end{array}\right] \xrightarrow[\frac{1}{3}R_3]{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & -1 & -2 \end{array}\right]$$

$$\xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -2 & -6 \end{array}\right] \xrightarrow{-\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

so the solution is $(1, 2, 3)$.

5 (20 Points). *Consider the system of equations*

$$\begin{aligned} x + y &= 1 \\ 2x - y + z &= 2 \\ -4x + y - 2z &= 3. \end{aligned}$$

Given

$$\begin{pmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ -4 & 1 & -2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -2 & -1 \\ -2 & -5 & -3 \end{pmatrix},$$

solve the system.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -2 & -1 \\ -2 & -5 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 + 4 + 3 \\ 0 - 4 - 3 \\ -2 - 10 - 9 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -7 \\ -21 \end{pmatrix}$$