4/15/16 (1)

$2x + 3y = 4$

$5x + 7y = 4$

a) $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

b) what is the inverse of A?

$$\frac{1}{14-15}\begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix} = -1\begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} = B$$

For a 2x2 matrix, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse is

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

c) Solve the system.

$$\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -16 \\ 12 \end{bmatrix}.$$

So the solution is $(-16, 12)$.

$$6\left(\frac{x}{2} + \frac{y}{3}\right) = (-5)6 \iff 3x + 2y = -30$$

$$3\left(\frac{x}{3} + y\right) = (-8)3 \qquad x + 3y = -24$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -30 \\ -24 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -30 \\ -24 \end{bmatrix}.$$

In general, for any real numbers a, b, c, d, e, f such that $ad - bc \neq 0$

The solution to the system

$$ax + by = e \qquad \longleftrightarrow \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$
$$cx + dy = f$$

is given by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} e \\ f \end{bmatrix}$$

Analogue: If a is any real number,

$$a \cdot a^{-1} = a \cdot \left(\frac{1}{a}\right) = \frac{a}{a} = 1.$$

2) $2x + 3y = 9$
$5x + 7y = 19.$

$$\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 19 \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \end{bmatrix}$$

3) $x - y + 7z = 4$
$x - y + 8z = 3$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 7 & 4 \\ 1 & -1 & 8 & 3 \end{array}\right]$$

reduced form $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 11 \\ 0 & 0 & 1 & -1 \end{array}\right] \longleftrightarrow \begin{aligned} x - y &= 11 \\ z &= -1 \end{aligned}$

~~x~~ $x - y = 11 \Rightarrow x - 11 = y$

Solutions: $\{(x, x-11, -1) \mid x \in \mathbb{R}\}$

By hand:

$$\left[\begin{array}{ccc|c} 1 & -1 & 7 & 4 \\ 1 & -1 & 8 & 3 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & -1 & 7 & 4 \\ 0 & 0 & 1 & -1 = 3-4 \end{array}\right]$$

$$\xrightarrow{R_1 - 7R_2} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 4-7(-1) \\ 0 & 0 & 1 & -1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -1 & 0 & 11 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

4) $2x - y = 0$

$x + y + z = 18$

$x - z = 2$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 1 & 1 & 1 & 18 \\ 1 & 0 & -1 & 2 \end{array}\right]$$

$$\xrightarrow[2R_3 - R_1]{2R_2 - R_1} \left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & 3 & 2 & 36 \\ 0 & 1 & -2 & 4 \end{array}\right] \xrightarrow[3R_3 - R_2]{3R_1 + R_2} \left[\begin{array}{ccc|c} 6 & 0 & 2 & 36 \\ 0 & 3 & 2 & 36 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$$\xrightarrow{\frac{1}{8}R_3} \left[\begin{array}{ccc|c} 6 & 0 & 2 & 36 \\ 0 & 3 & 2 & 36 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2-2R_3]{R_1-2R_3} \left[\begin{array}{ccc|c} 6 & 0 & 0 & 30 \\ 0 & 3 & 0 & 30 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad 5$$

$$\xrightarrow[\frac{1}{3}R_2]{\frac{1}{6}R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 3 \end{array}\right] \qquad (\cancel{3}5, 10, 3).$$

$$2x - y = 0$$

$$2(x + y + z) = 18 \cdot 2$$

$$\begin{aligned} & 2x + 2y + 2z = 36 \\ - \; & 2x - y + 0 \cdot z = 0 \\ \hline & 0 \quad 3y + 2z = 36 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \overset{A^{-1}}{\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}^{-1}} \overset{B}{\begin{bmatrix} 0 \\ 18 \\ 2 \end{bmatrix}} = \begin{bmatrix} 5 \\ 10 \\ 3 \end{bmatrix}$$