4/8/16 ①

b) ~~Find the optimal~~

If CTV notices that RTV is showing docudramas ~~documentaries~~ half the time and reality shows half the time, what would CTV's best strategy be?

Given $R = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$, let

$$C = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

Compute

$$e = RPC = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 & 2 \\ -1 & 1 & -1 & 2 \\ -2 & 0 & 0 & 1 \\ 3 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

$$= -\frac{3}{2}x + \frac{1}{2}y - \frac{1}{2}z + \frac{3}{2}t$$

~~Assume~~ ~~x = 0~~ and ~~z = 0~~

~~$e = \frac{1}{2}y + \frac{3}{2}t$~~ ~~$y + t = 1 \Rightarrow t = 1 - y$~~

~~$= \frac{1}{2}y + \frac{3}{2}(1 - y)$~~

~~$= \frac{1}{2}y + \frac{3}{2} - \frac{3}{2}y = -\frac{2}{2}y + \frac{3}{2} = -y + \frac{3}{2}$~~

$e = -\frac{3}{2}x + \frac{1}{2}y - \frac{1}{2}z + \frac{3}{2}t$

Want to minimize e because this represents the number of viewers CTV loses to RTV.

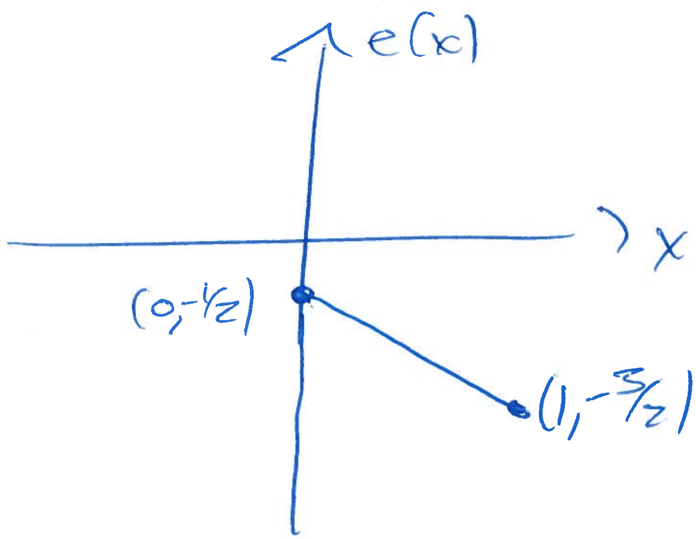
So we may assume that $y = t = 0$. Thus

$$x + z = 1 \Rightarrow z = 1 - x.$$

$$-\frac{3}{2}x - \frac{1}{2}z = e$$

$$\Rightarrow e = -\frac{3}{2}x - \frac{1}{2}(1-x)$$
$$= -\frac{3}{2}x - \frac{1}{2} + \frac{1}{2}x$$
$$= -\frac{2}{2}x - \frac{1}{2}$$
$$= -x - \frac{1}{2}.$$

Thus we have the graph of $e(x)$



Take $x = 1$

so $e(x) = -3/2$

i.e. CTV gains 1500 viewers.

The strategy is always show Nature Docs.

Minimax Criterion

A player using the minimax criterion chooses a strategy that, amongst all other strategies, minimizes the effect of the other player's best counterstrategy. That is, an optimal strategy according to the minimax criterion is one that minimizes the maximum damage the other player can do

Fundamental Principle of Game Theory

Each player tries to use its best possible strategy, and assumes the other player is doing the same.

Finding an optimal ~~solution~~ strategy for a game is called solving the game.

Consider the game

$$P = A \overset{\qquad B}{\begin{array}{c} \\ x \\ y \end{array} \begin{array}{c} \begin{array}{cc} u & v \end{array} \\ \begin{bmatrix} -2 & 0 \\ -3 & -1 \end{bmatrix} \end{array}}$$

Any strategy $R=[s\ t]$ we have

$$s+t=1 \Rightarrow s=1-t \text{ or } t=1-s$$

so $R=[s\ \ 1-s]$. Similary, we could always write the strategy for B as

$$C=\begin{bmatrix} w \\ 1-w \end{bmatrix}$$

where w is the percentage of times B plays u.

$$e = RPC = [s\ \ 1-s]\begin{bmatrix} -2 & 0 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} w \\ 1-w \end{bmatrix}$$

$$= [s\ \ 1-s]\begin{bmatrix} -2w \\ -3-1+w \end{bmatrix}$$

$$= [s\ \ 1-s]\begin{bmatrix} -2w \\ -4+w \end{bmatrix}$$

$$= -2sw + (1-s)(-4+w)$$

$$= -2sw - 4 + w + 4s - sw$$

$$= -3sw + 4s + w - 4.$$

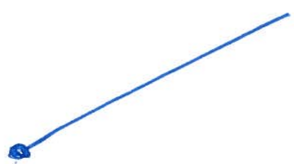
$$e(s,w) = -3sw + 4s + w - 4.$$

Given a value for $s = s_0$;

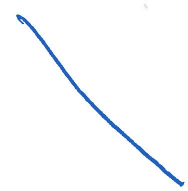
$$e(w) = e(s_0, w) = (-3s_0)w + w + (4s_0 - 4)$$
$$= (-3s_0 + 1)w + (4s_0 - 4)$$

Constants (i.e. just numbers)

And this is just a line. As player B, we want to minimize $e(w)$, which looks like one of



or



here take $w = 0$

here take $w = 1$.

So player B's best counter-strategy is pure:

either $w = 0$ and $1 - w = 1$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

or $w = 1$ and $1 - w = 0$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

This reduces the problem to solving a maximization problem.

Either we have

$$e = e(s, 1) = -3s + 4s + 1 - 4 = s - 3$$

or

$$e = e(s, 0) = -3s(0) + 4s + 0 - 4 = 4s - 4.$$