$$\begin{array}{cc|cc} & & B & \\ & & a & b \\ A & \begin{array}{c} p \\ q \end{array} & \multicolumn{2}{c}{\begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}} \end{array}$$

A: p 3/4 of time
q 1/4 " "

B: a 1/5 of time
b 4/5 " "

Play 100 games
4 possible outcomes
$\{(p,a), (p,b), (q,a), (q,b)\}$

Case 1: (p,a) – 15 times

Case 2: (p,b) – 60 times

Case 3: (q,a) – 5 times

Case 4: (q,b) – 20 – times

Each time case 1 occurs, A ~~wins~~ gains 3 points
Each time case 2 occurs A loses 1 point (gains −1 points)

Each time case 3 occurs A loses 2 points

Each time case 4 occurs A gains 3 points

In the end, A has

$$15 \cdot 3 + 60(-1) + 5(-2) + 20(3)$$

$$= 45 - 60 - 10 + 60 = 35$$

On average, A expects to win

$$e = 35/100 = 7/20$$

points per game.

<u>Observation</u>: This is tedious.

Record the strategy for A as a row matrix

$$R = \begin{bmatrix} 3/4 & \frac{1}{4} \end{bmatrix}$$

and record the strategy for B as a column matrix

$$C = \begin{bmatrix} 1/5 \\ 4/5 \end{bmatrix}$$

Then we can compute the expected payoff as

$$e = R\overset{\text{payoff matrix}}{P}C$$

$$= \begin{bmatrix} 3/4 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 1/5 \\ 4/5 \end{bmatrix}$$

$$= \begin{bmatrix} 3/4 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 3(1/5) + (-1)(4/5) \\ -2(1/5) + 3(4/5) \end{bmatrix}$$

$$= 3/4\left(3(1/5) + (-1)(4/5)\right) + \frac{1}{4}\left((-2)(1/5) + 3(4/5)\right)$$

$$= 3(3/4)(1/5) + (-1)(3/4)(1/5) + (-2)\left(\frac{1}{4}\right)\left(\frac{1}{5}\right) + 3\left(\frac{1}{4}\right)(4/5)$$

$$= \frac{1}{100}\left(\overset{\text{points}}{3}\underbrace{(3/4)(1/5)\cdot 100}_{\#\text{ times case 1 occurs}} + \overset{\text{points}}{(-1)}\underbrace{(3/4)(1/5)\cdot 100}_{\#\text{ times case 2 occurs}} + \overset{\text{points}}{(-2)}\underbrace{\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)100}_{\#\text{ times case 3 occurs}} + \overset{\text{points}}{3}\underbrace{\left(\frac{1}{4}\right)(4/5)\cdot 100}_{\#\text{ times case 4 occurs}} \right)$$

Eg: Rock, paper, scissors

$$
\begin{array}{cc}
 & \begin{array}{ccc} & B & \\ r & p & s \end{array} \\
A \begin{array}{c} r \\ p \\ s \end{array} & \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}
\end{array}
$$

Given strategies

A: r 50%, p 25%, s 25%

B: r 0%, p 100%, s 0%

Expected payoff:

$$e = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}.$$

We expect A to lose (on average) 1 out of every four games.

Say we play four games with these strategies. The outcomes are (r, p) – loss

(r, p) – loss

(p, p) – tie

(s, p) – win

$-1 + \cancel{1} + 0 + \cancel{1} = -1$

$\frac{-1}{4}$.

Solving a Game

E.g.:

CTV

		Nature Doc.	Symphony	Ballet	Opera
RTV	Sitcom	2	1	-2	2
	Docudrama	-1	1	-1	2
	Reality Show	-2	0	0	1
	Movie	3	1	-1	1

Each # indicates in thousands viewers gained by RTV.

a) If RTV notices that CTV is showing nature docs ½ of the time and symphonies the other half, what would RTV's best strategy be and how many viewers would it expect to gain?

$$e = [x \; y \; z \; t] \begin{bmatrix} 2 & 1 & -2 & 2 \\ -1 & 1 & -1 & 2 \\ -2 & 0 & 0 & 1 \\ 3 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$$

$$= [x \ y \ z \ t]\begin{bmatrix} 3/2 \\ 0 \\ -1 \\ 2 \end{bmatrix} = 3/2\,x - z + 2t$$

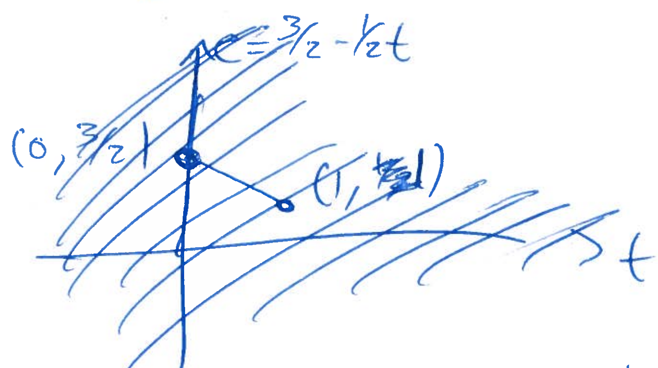
We want e to be as high as possible

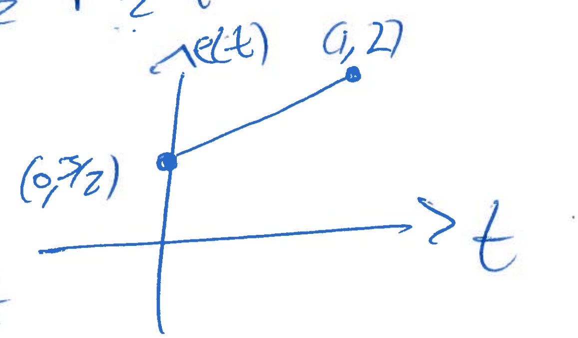
Note that $x+y+z+t=1$ and all of x, y, z, t are non-negative. Since playing reality shows only reduces ratings, we should assume $z=0$. We can also assume $y=0$. So we have

$$e = 3/2\,x + 2t$$

$$x + t = 1 \Rightarrow x = 1-t$$

$$\Rightarrow e = 3/2(1-t) + 2t$$
$$= 3/2 - 3/2\,t + \tfrac{4}{2}t = \tfrac{3}{2} + \tfrac{1}{2}t$$





This says we should choose $t=1$, ~~$x=1-t=1$~~. Best strategy for RTV is

$x = 1-t = 1-1 = 0$. $\quad x=y=z=0 \quad t=1$

I.e. always show Movies.