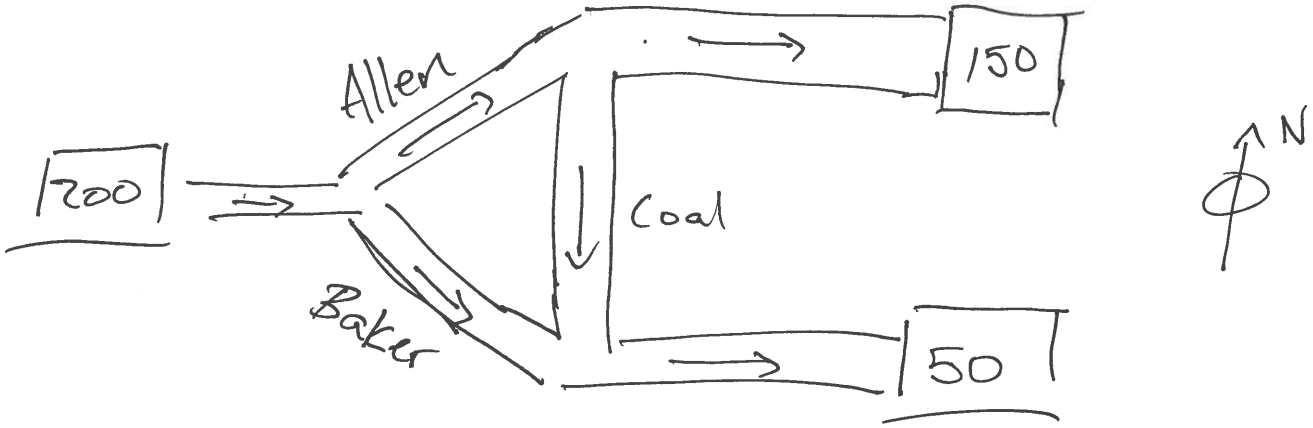
Eg. 3

4/1/16

Urbanville




200 entering on the West
150 exiting to the North
50 exiting to the South
} each hour

a. From this information, is it possible to determine how many cars drive along Allen, Baker, and Coal streets every hour?

Let
x = # of cars driving on Allen,
y = # of cars driving on Baker, and
z = # of cars driving on Coal
} each hour,

$$\begin{aligned} x + y &= 200 \\ x - z &= 150 \\ y + z &= 50 \end{aligned} \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 200 \\ 1 & 0 & -1 & 150 \\ 0 & 1 & 1 & 50 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 150 \\ 0 & 1 & 1 & 50 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} x - z &= 150 \\ y + z &= 50 \\ 0 &= 0 \end{aligned}$$

Dependent system, $z$ is arbitrary (almost!). We cannot determine how many cars drive along each street every hour.

b. What is the maximum possible traffic flow along Baker?

$$y + z = 50 \Rightarrow y = 50 - z$$

The maximum flow along baker is $y = 50$ when $z = 0$.

c. What is the minimum possible flow along Allen?

$$x - z = 150 \Rightarrow x = 150 + z$$

The minimum is $x = 150$ when $z = 0$.

d. What is the maximum flow along Coal?

$$0 \leq y = 50 - z$$

$$\Rightarrow z \leq 50.$$

Eg. 4 Rental Car Company has 4 locations SW, NE, SE, & NW.

NW has 20 more cars than it needs

NE has 15 more cars than it needs

SW needs 10 more than it has

SE needs 25 more than it has

Costs $10 to drive a car NW to SW

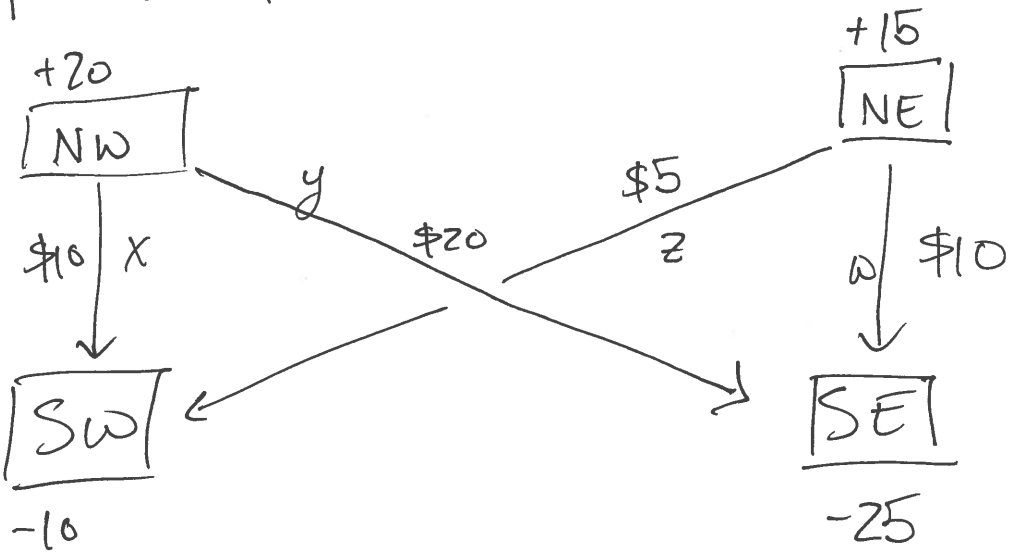
Costs $5 to drive a car NE to SW

Cost $10 to drive a car NE to SE

" $20 " " " NW to SE

The company will spend $475 to rearrange cars

How many cars will it drive from each of NW & NE to each of SW & SE?



$x$ = # cars driven NW to SW

$y$ = # cars driven NW to SE

$z$ = # cars driven NE to SW

$w$ = # cars driven NE to SE.

$$10x + 20y + 5z + 10w = 475$$
$$x + z = 10$$
$$y + w = 25$$
$$x + y = 20$$
$$z + w = 15$$

$$\left[\begin{array}{cccc|c} 10 & 20 & 5 & 10 & 475 \\ 1 & 0 & 1 & 0 & 10 \\ 0 & 1 & 0 & 1 & 25 \\ 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 1 & 15 \end{array}\right]$$

rref →

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x = 5$
$y = 15$
$z = 5$
$w = 10$