§3.2

Eg.:

$$-\tfrac{2}{5}x + y - \tfrac{3}{5}z = 0$$

$$-\tfrac{3}{5}x - \tfrac{2}{5}y + z = 0$$

$$x - \tfrac{3}{5}y - \tfrac{2}{5}z = 0$$

Solve If it is dependent, express your answer in terms of x, where $y = y(x)$ and $z = z(x)$

$$\begin{aligned} -2x + 5y - 3z &= 0 \\ -3x - 2y + 5z &= 0 \\ 5x - 3y - 2z &= 0 \end{aligned} \quad \leftrightarrow \left[\begin{array}{ccc|c} -2 & 5 & -3 & 0 \\ -3 & -2 & 5 & 0 \\ 5 & -3 & -2 & 0 \end{array}\right]$$

Reduced Row Echelon Form of this matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This is called "dependent."

$$\updownarrow$$

$$\begin{aligned} x - z &= 0 \\ y - z &= 0 \end{aligned} \iff \begin{aligned} x &= z \\ y &= z \end{aligned} \qquad \begin{aligned} &(z, z, z) \\ &\| \\ &(x, x, x) \end{aligned}$$

Check: (x, x, x) is a solution to my original system ②

$$-2x + 5x - 3x = x(-2+5-3) = 0$$
$$-3x - 2x + 5x = x(-3-2+5) = 0$$
$$5x - 3x - 2x = x(5-3-2) = 0$$

E.g.: $\begin{aligned} x - y + 7z &= 4 \\ x - y + 8z &= 3 \end{aligned} \longleftrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 7 & 4 \\ 1 & -1 & 8 & 3 \end{array}\right]$

Express answer in terms of x.

After Gauss-Jordan Elim. get

Dependent $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 11 \\ 0 & 0 & 1 & -1 \end{array}\right] \longleftrightarrow \begin{aligned} x - y &= 11 \\ z &= -1 \end{aligned}$

$y = x - 11$, ① $(x, x-11, -1)$. ← This the answer Web Assign wants

(Web Assign will mark this wrong. →) ② $(y+11, y, -1)$

In ①, choose an x value, get a solution to the system (x is the independent variable)

In ~~②~~ ②, choose a y value, get a solution to the system (y is the independent variable).

E.g. (2) Airline purchases

A330 -300s	320 Pass	$200 mil
767 -200ERs	250 Pass	$125 mil
787 -9s	275 Pass	$200 mil

to ~~meet~~ meet demand for 4,800 seats. The airline bought twice as many 787s as 767s for a total cost of $3.1 billion or $3,100 million. How many of each did they buy?

Let $x = \#$ A330s

$y = \#$ 767s

$z = \#$ 787s

$$4800 = 320x + 250y + 275z$$

$$3{,}100 = 200x + 125y + 200z$$

$$z = 2y \Leftrightarrow -2y - z = 0$$

$$\left[\begin{array}{ccc|c} 320 & 250 & 275 & 4800 \\ 200 & 125 & 200 & 3100 \\ 0 & 2 & -1 & 0 \end{array}\right]$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 8 \end{bmatrix} \qquad \begin{aligned} x &= 5 \\ y &= 4 \\ z &= 8 \end{aligned}$$

Check:

$$\begin{bmatrix} 320 & 250 & 275 \\ 200 & 125 & 200 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 4800 \\ 3100 \\ 0 \end{bmatrix} \checkmark$$