Examples



$$\begin{aligned} -x + 2y - z &= 0 \\ -x - y + 2z &= 0 \\ 2x + 0y - z &= 4 \end{aligned} \quad \longleftrightarrow \quad \left[\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ 2 & 0 & -1 & 4 \end{array}\right]$$

$$\xrightarrow[R_3 + 2R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 4 & -3 & 4 \end{array}\right]$$

$$\xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 4 & -3 & 4 \end{array}\right]$$

$$\xrightarrow[R_3 + 4 \cdot R_2]{R_1 + 2R_2} \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right] \qquad (4, 4, 4).$$

$$\xrightarrow[R_2 - R_3]{R_1 - R_3} \left[\begin{array}{ccc|c} -1 & 0 & 0 & -4 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow[-R_2]{-R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$\begin{aligned} x + y + z &= -1 \\ 2x + 2y + 2z &= 2 \\ \tfrac{3}{5}x + \tfrac{3}{5}y + \tfrac{3}{5}z &= \tfrac{2}{5} \end{aligned} \quad \longleftrightarrow \quad \left[\begin{array}{ccc:c} 1 & 1 & 1 & -1 \\ 2 & 2 & 2 & 2 \\ 3/5 & 3/5 & 3/5 & 2/5 \end{array}\right]$$

$$\xrightarrow[5\cancel{3/5}R_3]{\frac{1}{2}R_2} \left[\begin{array}{ccc:c} 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 2 \end{array}\right]$$

$$\xrightarrow[R_3 - 3R_1]{R_2 - R_1} \left[\begin{array}{ccc:c} 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 5 \end{array}\right] \quad \longleftrightarrow \quad \begin{aligned} x + y + z &= -1 \\ 0x + 0y + 0z &= 2 \\ 0x + 0y + 0z &= 5. \end{aligned}$$

$0 = 2 \qquad 0 = 5$

This is absurd.

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$$\begin{aligned} x + y + z &= -1 \\ x + y + z &= \cancel{2}1 \\ 3x + 3y + 3z = 2 \iff x + y + z &= 2/3 \end{aligned}$$

<u>No Solutions</u>

In particular, these are three parallel planes.

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$$\begin{aligned} x + y - z &= -2 \\ x - y - 7z &= 0 \\ \tfrac{2}{7}x \quad - \tfrac{8}{7}z &= 14 \end{aligned} \quad \longleftrightarrow \quad \left[\begin{array}{ccc:c} 1 & 1 & -1 & -2 \\ 1 & -1 & -7 & 0 \\ 2/7 & 0 & -8/7 & 14 \end{array}\right]$$

$$\xrightarrow{7R_3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 1 & -1 & -7 & 0 \\ 2 & 0 & -8 & 98 \end{array}\right]$$

$$\xrightarrow[R_3-2R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -2 & -6 & 2 \\ 0 & -2 & -6 & 102 \end{array}\right]$$

$$\xrightarrow[\frac{1}{2}R_3]{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -1 & -3 & 1 \\ 0 & -1 & -3 & 51 \end{array}\right]$$

$$\xrightarrow{R_3-R_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -1 & -3 & 1 \\ 0 & 0 & 0 & 50 \end{array}\right]$$

The last row is equivalent to the equation

$$0 = 0x + 0y + 0z = 50$$

and clearly $0 \neq 50$. No solutions.

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$$\begin{aligned} x - y &= 0 \\ x + y &= 0 \\ x &= 0 \end{aligned} \quad \longleftrightarrow \quad \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right]$$

$$R_1 \leftrightarrow R_3 \quad \longrightarrow \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 - R_1]{R_2 - R_1} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 + R_2} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x = 0$

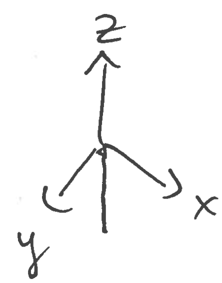
$y = 0$

$0x + 0y + 0z = 0$

$0 = 0$.

The solution to this system is the vertical line in 3-space (which is the z-axis)

$$\{(0,0,z) \mid z \in \mathbb{R}\}.$$

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Given a 3x3 matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

the determinant is

$$a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31})$$

+ [illegible]

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$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$1\left(\det\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right) - (-1)\cdot\det\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} + 0\det\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\| $$

$$0.$$

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$f(x) = x^2 + 2$, no inverse (fails horizontal line test)

$g(x) = x + 5$, $g^{-1}(x) = x - 5$.