$-\frac{2}{3}x + \frac{1}{2}y = -3$

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$\frac{1}{4}x - y = \frac{11}{4}$

Step 0: Check the determinant.

$$\left[\begin{array}{cc|c} -2/3 & \frac{1}{2} & -3 \\ \frac{1}{4} & -1 & 11/4 \end{array}\right]$$

$$-2/3(-1) - \left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}\right) = \frac{2}{3} - \frac{1}{8} \neq 0.$$

$$= \frac{16}{24} - \frac{3}{24} = \frac{13}{24}.$$

Step 1: Clear fractions from the matrix.

Multiply row 1 (R_1) by 6, multiply R_2 by 4

$$\left[\begin{array}{cc|c} -4 & 3 & -18 \\ 1 & -4 & 11 \end{array}\right]$$

Step 2: Designate the first non-zero entry in the first row as a pivot; this entry is used to clear (i.e. make zero) the rest of the column.

Add R_1 to $4 \cdot R_2$: $(4R_2 + R_1)$

Step 3: Clear column.

$$\left[\begin{array}{cc|c} -4 & 3 & -18 \\ 4\cdot(1)+(-4) & =4\cdot 4+3 & 4\cdot 11+(-18) \end{array}\right]$$

$$=$$

$$\left[\begin{array}{cc|c} -4 & 3 & -18 \\ 0 & -13 & 26 \end{array}\right]$$

Simplification Step (optional): Get rid of common factors in a row.

Multiply the second row (R_2) by $\frac{1}{13}$.

$$\left[\begin{array}{cc|c} -4 & 3 & -18 \\ 0 & -1 & 2 \end{array}\right]$$

Step 4: Select the first non-zero number in the second row as a pivot.

Step 5: Clear the second column as in step 3.

$R_1 + 3R_2$

$$\left[\begin{array}{cc|c} -4+3\cdot 0 & 3+3(-1) & -18+3(2) \\ 0 & -1 & 2 \end{array}\right]$$

$$=$$

$$\left[\begin{array}{cc|c} -4 & 0 & -12 \\ 0 & -1 & 2 \end{array}\right]$$

4

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right]$$

$$\xrightarrow[R_3 - R_1]{R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 0 & 3-3(-1) & -1-3\cdot5 & 10-3(-6) \\ 0 & 3-(-1) & 2-5 & 5-(-6) \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 0 & 6 & -16 & 28 \\ 0 & 4 & -3 & 11 \end{array}\right]$$

$$\xrightarrow[\frac{1}{2}R_2]{} \left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 0 & 3 & -8 & 14 \\ 0 & 4 & -3 & 11 \end{array}\right]$$

$$\xrightarrow[3R_3 - 4R_2]{3R_1 + R_2} \left[\begin{array}{ccc|c} 3 & 0 & 7 & -4 \\ 0 & 3 & -8 & 14 \\ 0 & 0 & 23 & -23 \end{array}\right]$$

$-9-4(-8)$

$-9+32=23$

$33-4\cdot14$

$33-56=-23$

Final Step: Simplify

$-\frac{1}{4}R_1$ and $-1R_2$

$$\left[\begin{array}{cc|c} -4\cdot -\frac{1}{4} & 0\cdot -\frac{1}{4} & -12\cdot -\frac{1}{4} \\ 0\cdot -1 & -1\cdot -1 & 2\cdot -1 \end{array}\right]$$

$$=$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right]$$

The Augmented Matrix is the system

$$1\cdot x + 0\cdot y = 3 \iff x = 3$$

$$0\cancel{6}\cdot x + 1\cdot y = -2 \iff y = -2.$$

$$-\frac{2}{3}(3) + \frac{1}{2}(-2) = -2 - 1 = -3.$$

$$\frac{1}{4}(3) - (-2) = \frac{3}{4} + 2 = \frac{3}{4} + \frac{8}{4} = \frac{11}{4}.$$

E.g.:

$$x - y + 5z = -6$$

$$3x + 3y - z = 10$$

$$x + 3y + 2z = 5$$

$$\xrightarrow{\frac{1}{23}R_3} \left[\begin{array}{ccc|c} 3 & 0 & 7 & -4 \\ 0 & 3 & -8 & 14 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\underset{R_1 - 7R_3}{\xrightarrow{R_2 + 8R_3}} \left[\begin{array}{ccc|c} 3 & 0 & 0 & -4-7(-1)=3 \\ 0 & 3 & 0 & 14+8(-1)=6 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\underset{\frac{1}{3}R_2}{\xrightarrow{\frac{1}{3}R_1}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Solution: $(1, 2, -1)$