Examples

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$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} c_{11} \\ c_{21} \end{pmatrix} = \begin{pmatrix} 17 \\ 39 \end{pmatrix}$$

$$\underset{\substack{| \quad\quad |\\ \text{rows} \quad \text{columns}}}{2 \times 2} \qquad \underset{\substack{| \quad\quad |\\ \text{rows} \quad \text{column}}}{2 \times 1} \quad = \quad 2 \times 1$$

$$c_{11} = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = 1 \cdot 5 + 2 \cdot 6 = 5 + 12 = 17$$

$$c_{21} = \begin{pmatrix} 3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = 3 \cdot 5 + 4 \cdot 6 = 15 + 24 = 39$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & \cancel{34}\ 50 \end{pmatrix}$$

$$(2 \times 2)(2 \times 2) = (2 \times 2)$$

$$c_{11} = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix} = 19 \qquad c_{12} = \begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 6 \\ 8 \end{pmatrix} = 22$$

$$c_{21} = \begin{pmatrix} 3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix} \qquad c_{22} = \begin{pmatrix} 3 & 4 \end{pmatrix}\begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$= 15 + 28 \qquad = 18 + \cancel{38}\ 32$$

$$= 43 \qquad = \cancel{84}\ 50$$

Systems

$$a_{11}x + a_{12}y = c_{11}$$
$$a_{21}x + a_{22}y = c_{21}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{pmatrix} = \begin{pmatrix} c_{11} \\ c_{21} \end{pmatrix}$$

coefficient matrix

$$\begin{aligned} 2x + y &= 5 \\ 3x + y &= 7 \end{aligned} \quad \longleftrightarrow \quad \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$\left.\begin{aligned} y &= -2x + 5 \\ y &= -3x + 7 \end{aligned}\right\} \Rightarrow -2x+5 = -3x+7$$

$$\Rightarrow x = 7-5 = 2.$$

$$\Rightarrow y = (-2)(2)+5 = -4+5 = 1.$$

Have a solution to the system, which is

$(2, 1)$

But we can see that

$$\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2\cdot 2 + 1\cdot 1 \\ 3\cdot 2 + 1\cdot 1 \end{pmatrix} = \begin{pmatrix} 4+1 \\ 6+1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

Given a system

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\underline{X}} = \underbrace{\begin{pmatrix} c_{11} \\ c_{21} \end{pmatrix}}_{C}$$

$$A\underline{X} = C$$

$$5x = 10 \Rightarrow x = 10/5$$

Want to say is if $A\underline{X} = C$, then

$$\underline{X} = C/A.$$

What is C/A? I.e. how do we "divide" matrices?

<u>Def</u>: The $m \times n$ matrix (for any m, n) with entries all zero is "the" zero matrix, I will denote this by 0.

If M is an $m \times n$ matrix and 0 is the $n \times k$ matrix of all zeroes, then

$$M \cdot 0 = 0 \leftarrow m \times k \text{ entries, all zero.}$$

E.g.:

$$\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 5 \\ 7 & 11 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$(m \times n)(n \times k) = m \times k$$

Non-Example

$$\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \leftarrow \text{not defined.}$$

Identity Matrix: The $n \times n$ identity matrix is the matrix

$$I_{n \times n} = \overbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}^{n} \Big\} n$$

and satisfies

$$I_{n \times n} M = M I_{n \times n} = M$$

for M an $n \times n$ Matrix.

E.g.:
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1\cdot1+\cancel{2\cdot0} & \cancel{1\cdot0}+2\cdot1 \\ 3\cdot1+\cancel{4\cdot0} & \cancel{3\cdot0}+4\cdot1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Analogue: If $a \in \mathbb{R}$, the multiplicative inverse of a is the number $u \in \mathbb{R}$ such that

$$a \cdot u = 1 = u \cdot a.$$

You know the number u as $1/a$.

Defn: Say an $n \times n$ matrix M is invertible if there is an $n \times n$ matrix N such that

$$N \cdot M = I_{n \times n} = M \cdot N.$$

If we want to solve a system

$$a_{11}x + a_{12}y = c_1$$
$$a_{21}x + a_{22}y = c_2$$

It's enough to find the inverse of the coefficient matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

For then we can solve the equation

$$AX = C$$

by multiplying by A^{-1} (the inverse ~~ons~~ of A) on the left:

$$A^{-1}(AX) = A^{-1}C$$

$$\parallel$$

$$(A^{-1}A)X$$

$$\parallel$$

$$I_{n\times n}X = X \; ,$$