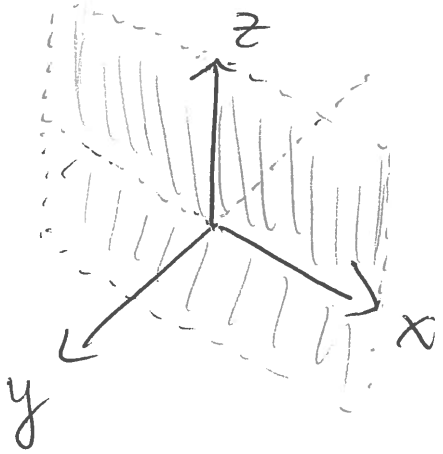
$y=0$

$$\{(x,0,z) \mid x \in \mathbb{R}, z \in \mathbb{R}\}$$



Def^{n}: A linear equation in 3 variables has the form

$$a_1x + b_1y + cz = d, \quad \text{not all of } a, b, c \text{ are } 0.$$

A linear equation in n variables has the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b, \quad a_i \in \mathbb{R}$$

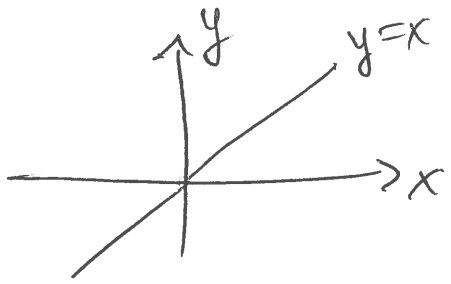
not all the a_i are zero.

Eg: (3 variables) $0 \cdot x + 0 \cdot y + z = 0 \quad (z=0)$

$x + 0 \cdot y + 0 \cdot z = 0 \quad (x=0)$

$0x + y + 0 \cdot z = 0 \quad (y=0)$

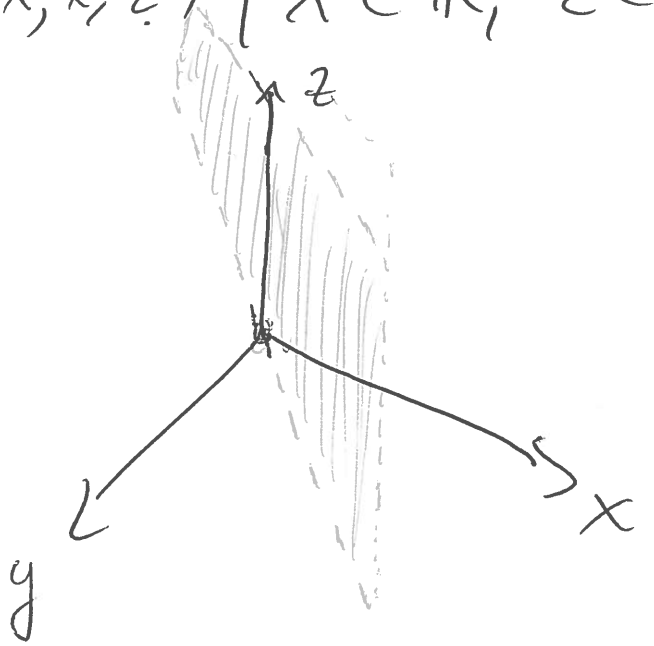
$y=x$ in $\mathbb{R}^2$



The graph of $y=x$: $\{(x,x) \mid x \in \mathbb{R}\} \subseteq \mathbb{R}^2$

The graph of $y=x$ in $\mathbb{R}^3$ is

$$\{(x,x,z) \mid x \in \mathbb{R},\ z \in \mathbb{R}\}$$



Eg: What are the simultaneous solutions to the system (in $\mathbb{R}^3$)

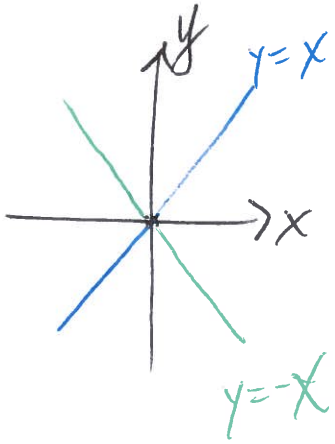
$$y = x$$

$$y = -x.$$

In $\mathbb{R}^2$, we know these intersect at one point, namely

$$x = -x$$
$$\Rightarrow 2x = 0$$
$$\Rightarrow x = 0$$
$$\Rightarrow y = 0$$

$(0, 0)$



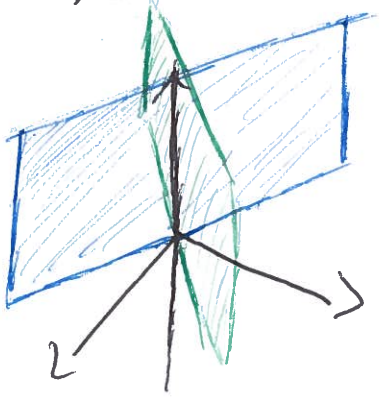
In $\mathbb{R}^3$ the simultaneous solutions are the points in

$$\{(x, x, z) \mid x \in \mathbb{R},\ z \in \mathbb{R}\} \cap \{(x, -x, z) \mid x \in \mathbb{R},\ z \in \mathbb{R}\}$$

graph of $y = x$ in $\mathbb{R}^3$ ∩ graph of $y = -x$ in $\mathbb{R}^3$

$$= \{(0, 0, z) \mid z \in \mathbb{R}\} \quad \text{(line)}$$

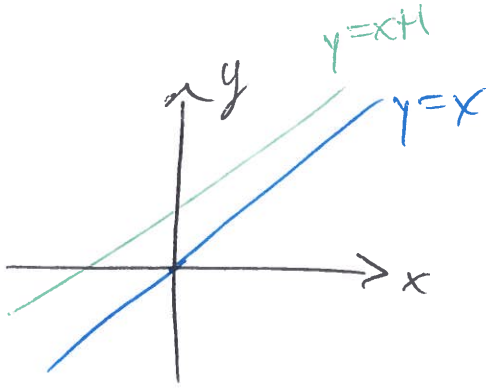
↖ z-axis



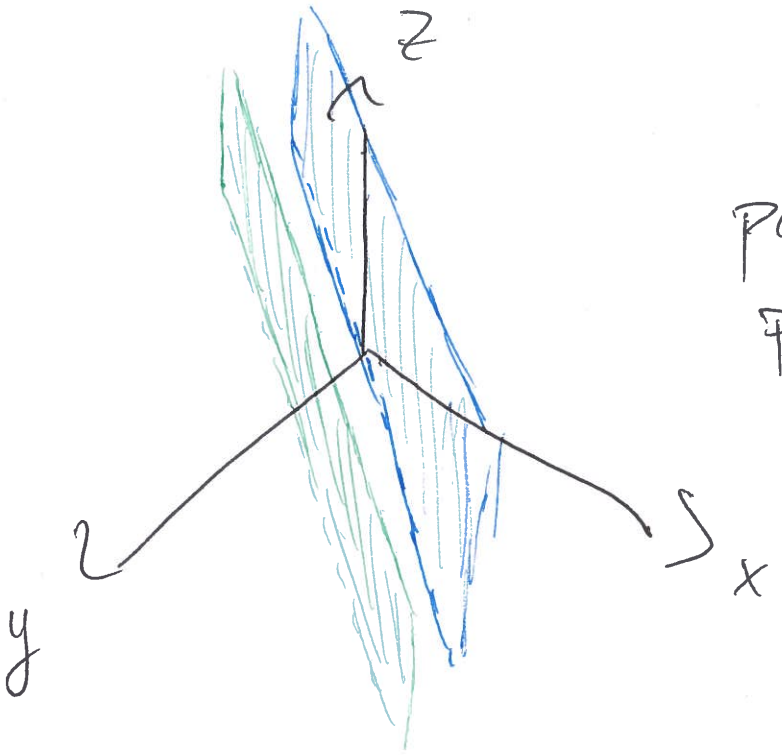
Eg: $y = x$
$y = x + 1$

in $\mathbb{R}^2$

parallel, no solution.



in $\mathbb{R}^3$



parallel planes, no simultaneous solutions.

Eg: $y = x$

$2y = 2x$

in $\mathbb{R}^2$, these are the same intersect in the line $y = x$

in $\mathbb{R}^3$, these are the same, intersect in the plane $y = x$.

Eg: System of 3 equations in 3 unknowns.

$y = x$

$y = -x$

$z = 3$

$$y = x = -x$$

$$\Rightarrow x + x = 0$$

$$\Rightarrow 2x = 0$$

$$\Rightarrow x = 0 = y.$$

There is one solution: $(0, 0, 3)$.

E.g.: $y = x$
$y = x + 1$
$z = 3$

This has no solutions.

$$x + 1 = y = x$$

$$x + 1 = x$$

$$x - x + 1 = 0$$

$$0 + 1 = 0$$

$$1 = 0. \quad \leftarrow \text{Nonsense.}$$