E.g.:

$y + x = 0 \iff y = -x$

$y - x = 0 \iff y = x$

A solution to this system is the point $(0,0)$

$0 + 0 = 0$

$0 - 0 = 0$.

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Solutions to a system of 2 linear equations in 2 unknowns are the points of intersection (of the two lines)

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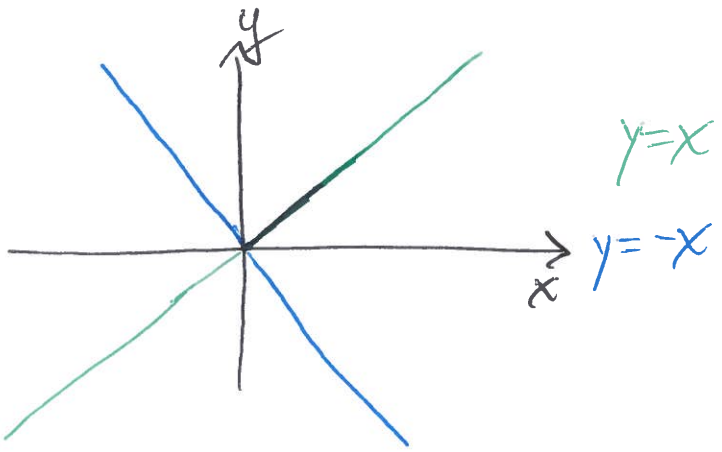
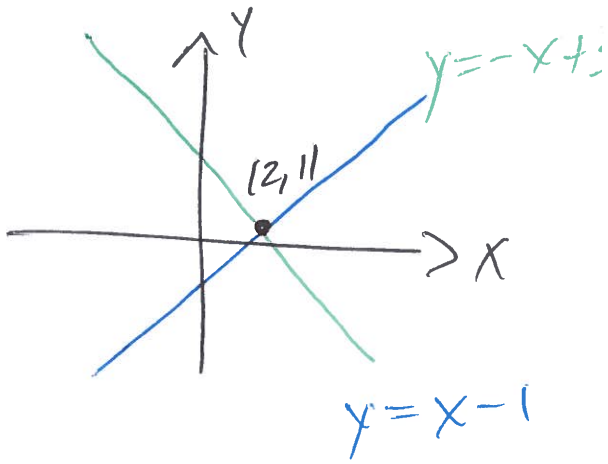
E.g: $y + x = 0$

$y - x = 0$



E.g.: Find all solutions to the system

$$x+y=3 \iff y=-x+3$$
$$x-y=1 \iff y=x-1$$




$$\Rightarrow x-1=-x+3$$
$$\Rightarrow 2x-1=3$$
$$\Rightarrow 2x=4$$
$$\Rightarrow x=4/2=2.$$

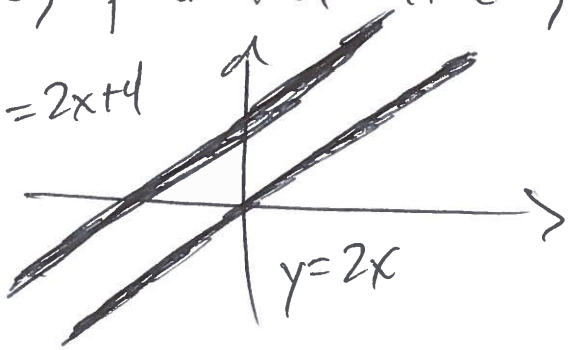
$$\Rightarrow y=2-1=1 \quad \text{or} \quad y=-2+3=1.$$

There is exactly one solution, $(2,1)$.

E.g: Find the solutions to the system

$$y-2x=4 \iff y=2x+4$$
$$y-2x=0 \iff y=2x$$

Same slope; parallel lines; no solutions.



$$2x+4=2x$$

$$\Rightarrow 2x+4-2x=2x-2x$$

$$\Rightarrow 4=0$$

This is absurd. No point of intersection.

E.g.: Find the solutions to

$$y-2x=4 \iff y=2x+4$$

$$2y-4x=8 \iff 2y=4x+8$$

$$\iff y=2x+4$$

These are the same line! These intersect in infinitely many points.

Algebraically,

$$2x+4=2x+4$$

$$\Rightarrow 2x+4-(2x+4)=2x+4-(2x+4)$$

$$\Rightarrow 0=0$$

This says these are the same line.

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These are all the possible solution types for 2 linear equations in 2 unknowns: intersect in 1 point (slopes not the same); don't intersect (parallel)

or they are the same line

E.g: $3x+5y=0$
$2x+7y=1$

$3x+5y=0 \iff 5y=-3x \iff y=-3/5\,x$

$2x+7y=1 \iff 7y=-2x+1 \iff y=-\frac{2}{7}x+\frac{1}{7}$.

$-3/5\,x=-\frac{2}{7}x+\frac{1}{7}$

$\Rightarrow -3/5\,x+\frac{2}{7}x=\frac{1}{7}$

$\Rightarrow 35\left(\frac{-3}{5}x+\frac{2}{7}x\right)=\frac{35}{7}=5$

$\Rightarrow -21x+10x=5$

$\Rightarrow -11x=5$

$\Rightarrow x=-5/11.$

$y=-\frac{3}{5}\left(\frac{-5}{11}\right)=\frac{3}{11}.$

$(-5/11, \frac{3}{11})$.

$35=5\cdot 7$

$35\left(\frac{-3}{5}\right)x+35\left(\frac{2}{7}\right)x$

$\frac{35}{5}=\frac{5\cdot 7}{5}=7.$

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$30\left(\frac{2}{5}x+\frac{3}{2}\right)=\left(\frac{1}{3}\right)30$

$30=5\cdot 6$
$30=15\cdot 2$

$\Rightarrow \frac{2\cdot 30}{5}x+\frac{3}{2}30=10 \Rightarrow 2\cdot 6x+3\cdot 15=10$
$\Rightarrow 12x+45=10$

Eg: A medieval alchemist's love potion calls for a number of eyes of newt and toes of frog, the total being 20, but with twice as many eyes of newt as toes of frog. How many of each is required?

Let $x$ be the number of eyes of newt

Let $y$ be the number of toes of frog.

Given $x+y=20$

$2y=x$

$\Rightarrow 2y+y=20$

$\Rightarrow 3y=20$

$\Rightarrow y=\frac{20}{3}$

$\Rightarrow x=2y=2\left(\frac{20}{3}\right)=\frac{40}{3}$.