

Recall

2/15/16

①

Let P, Q be propositions, i.e. statements that are either true or false (but neither both or none of those)

Conjunction/And

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

" P and Q "

Disjunction/or

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

" P or Q "

Eg: $P =$ "it is warm"

$Q =$ "it is sunny."

(2)

Conjunction: $P \wedge Q$

"It is warm and sunny."

The truth values for this statement are basically answers to the question "is this true?"

Today it is not warm and today is not sunny, so this statement is false today.

- Say the temperature ~~is~~ is 30°F and it is sunny. It is not warm, hence $P \wedge Q$ is false
- Say the temperature is 80°F and it's raining. It's not sunny, so $P \wedge Q$ is false
- Say the temperature is 80°F & it's sunny. Then $P \wedge Q$ is true

Disjunction: "It is warm or sunny." (3)

Today, it is not warm, it is not sunny, $p \vee q$ is false

- The temp. is 80°F and it's raining.
 p is true, so the $p \vee q$ is true.
- The temp is 30°F and the sun is out.
 p is false, q is true
 $p \vee q$ is true.
- The temp is 80°F , and it's Sunny.
 p is true, q is true,
 $p \vee q$ is true

E.g.: What is the truth value of

$$p \vee \neg p (\equiv T)$$

for p any proposition?

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

E.g.: What is the truth value of (9)

$$P \wedge \neg P?$$

This is false.

E.g.: $P =$ "It is warm."

$\neg P =$ "It is not warm."

"It is warm and not warm"
is always false. (contradiction)

"It is warm or not warm"
is always true (tautology) -

E.g.: Say S is the set of all students at USC. Say A is the set of all students taking Math 170.

What does the set $(S/A) \cup A$ represent?

$S \setminus A$ is the set of students ⑤
not taking Math 170.

A is the set of students taking Math
170

$(S \setminus A) \cup A$ is the set of students either
taking Math 170 or not taking
Math 170.

This is the set of all students at
USC, S .

$$\begin{aligned}(S \setminus A) \cup A &= \{x \in S \mid x \in S \setminus A \text{ or } x \in A\} \\ &= \{x \in S \mid x \notin A \text{ or } x \in A\} \\ &= \{x \in S \mid \neg(x \in A) \vee (x \in A)\} \\ &= \{x \in S\} = S.\end{aligned}$$

What does the set

(6)

$$(S \setminus A) \cap A$$

represent? This is the empty set.

$(S \setminus A) \cap A$ is the set of all students taking Math 170 and not taking Math 170. No students satisfy this condition.

$$\begin{aligned}(S \setminus A) \cap A &= \{s \in S \mid s \in (S \setminus A) \text{ and } s \in A\} \\ &= \{s \in S \mid \underline{s \notin A} \wedge \underline{s \in A}\} \\ &= \emptyset.\end{aligned}$$

A, B sets, ~~n~~ $|A| = m$, $|B| = n$

$$|A \times B| = mn.$$

E.g. $A = \{T, F\}$, $B = \{T, F\}$, $|A \times B| = |\{(T, T), (T, F), (F, T), (F, F)\}| = 4.$