2/12/16

No Motorcycle and Moped Parking

logically equivalent to

No Motorcycle & no moped park

$\Rightarrow$ go ahead and park anything here that isn't both a motorcycle & a moped

Eg.: Let S be the set of all students at USC.

Let A be the set of all students at USC taking Math 170

Let B be the set of all students at USC ~~Fall~~ majoring in Business.

What does the set $A \cup B$ represent?

This is the set of students at USC who

are either taking Math 170 or who are majoring in business (or both).

What does the set $S \setminus A$ represent?

This is the set of students at USC who are not taking 170.

What does the set $(S \setminus A) \cap B$ represent?

The set of students who are not taking Math 170 and are majoring in business.

What does the set $A \cap (S \setminus B)$ represent?

The set of students taking Math 170 and not majoring in business.

What does the set

$$[(S \setminus A) \cap B] \cup [A \cap (S \setminus B)]$$

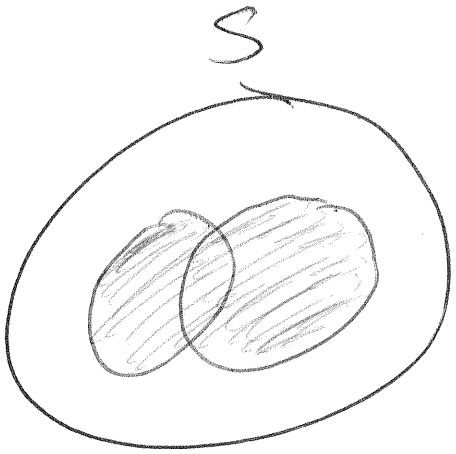
represent?

The set of students who are either

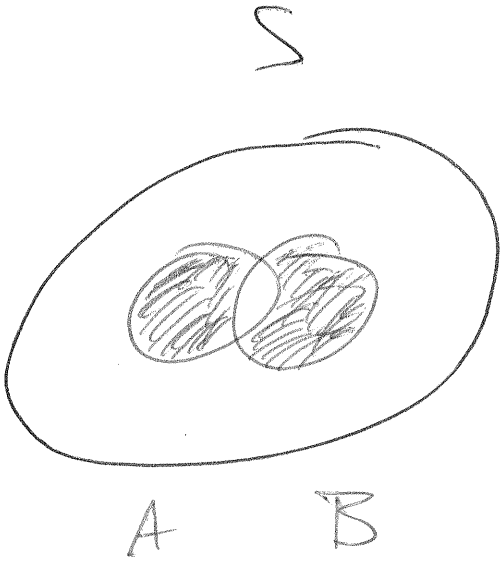
(i) & not taking Math 170 & majoring in business (3)

(ii) are taking Math 170 & not majoring in business.

~~The~~ Another way to phrase this is as the set of students who are either taking Math 170 or majoring in business, but not both.



$A \cup B$



$[(S \setminus A) \cap B] \cup [A \cap (S \setminus B)]$

"Disjoint Union"

7 contestants C - set of contestants.

4 semi-finalists

How many contain neither Ben nor Ann?

$S = \{Ben, Ann\}$

We want draw up a list of semi finalists from the set $C \setminus S$.

$$|C \setminus S| = 5$$

There $\binom{5}{4}$ ways to select the semi-finalists.

$C(5,4)$

$$C(5,4) = \binom{5}{4} = \frac{5!}{(5-4)!\,4!} = \frac{5!}{4!} = \frac{5(4!)}{4!} = 5$$

Check: $C = \{c_1, c_2, c_3, c_4, c_5, Ben, Ann\}$, $S = \{Ben, Ann\}$

$C \setminus S = \{c_1, c_2, c_3, c_4, c_5\}$

c_1	c_1	c_1	c_1	c_2
c_2	c_2	c_2	c_3	c_3
c_3	c_3	c_5	c_4	c_4
c_4	c_5	c_4	c_5	c_5

7 contestants, C = set of contestants

3 prizes

How many ways can you choose the winners such that Ben & Ann lose?

This is the number of 3 permutations of the 5-element set $C \setminus \{Ann, Ben\}$.

$$P(5,3) = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3!}{3!} = 20$$

An r-combination of a set of n elements is a subset of r elements.

$$\{1, 2, 3, 4, \ldots, r\} = \{r, r-1, \ldots, 4, 3, 2, 1\}$$

* Permutations counts the # of ordered lists
* Combinations counts the # of unordered lists/subsets

De Morgan's Laws

Show $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T