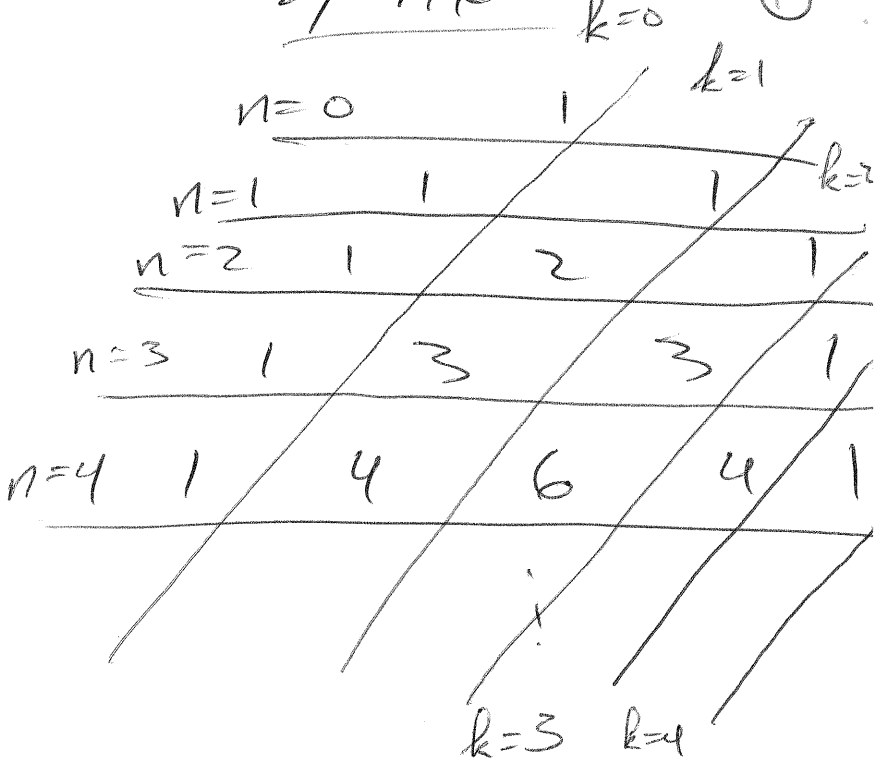
$\binom{0}{0}$

$\binom{1}{0}$ $\binom{1}{1}$

$\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

$\vdots$



For any row, the sum of the binomial coefficients is 2^n

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n}.$$

Pf: $2^n = (1+1)^n$

$$= \binom{n}{0} 1^n 1^0 + \binom{n}{1} 1^{n-1} 1^1 + \cdots + \binom{n}{n-1} 1^1 1^{n-1} + \binom{n}{n} 1^n$$

$$= \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n-1} + \binom{n}{n}. \blacksquare$$

Recall: $(x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \cdots + \binom{n}{n-1} x^1 y^{n-1} + \binom{n}{n} y^n$

6.1

$A = \{$June, Janet, Jill, Justin, Jeff, Jello$\}$

$B = \{$Janet, Jello, Justin$\}$

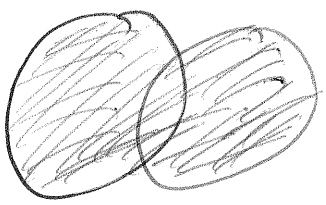
$C = \{$Sally, Solly, Molly, Jolly, Jello$\}$.

$A \cup B = A$

$A \cup B$

$\{$June, Janet, Jill, Justin, Jeff, Jello$\}$.

$A \cup C = \{$June, Janet, Jill, Justin, Jeff, Jello, Sally, Solly, Molly, Jolly$\}$



~~$A \cup B$~~ $A \cup C$

$B \cup C = \{$Janet, Jello, Justin, Sally, Solly, Molly, Jolly$\}$.

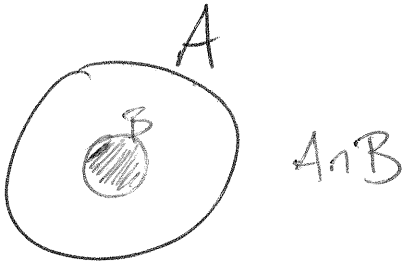
$B \cup C$

$A \cup B = \{x \in A \text{ or } x \in B\}$

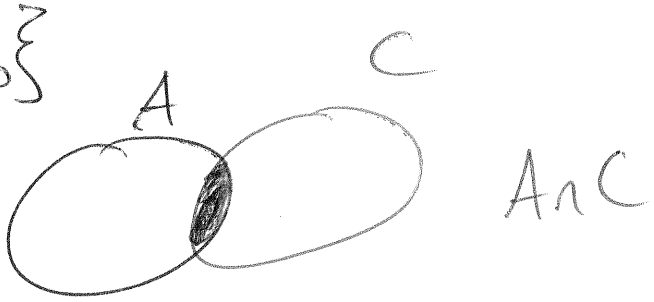
$x \in A \cup B \iff x \in A$ or $x \in B$.

$x \in A \cap B \iff x \in A$ and $x \in B$.

$A \cap B = \{\text{Janet, Jello, Justin}\} = B$.



$A \cap C = \{\text{Jello}\}$

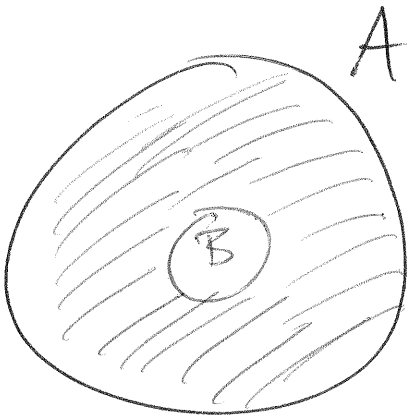


$A \cap C = \emptyset$

$A = \{1, 2, 3\}$ $C = \{4, 5, 6\}$

$$A \setminus B = \{x \in A \mid x \notin B\}$$

$$= \{\text{June}, \text{Jill}, \text{Jeff}\}$$



$A \setminus B$

$$S = \{\text{June}, \text{Janet}, \text{Jill}, \text{Justin}, \text{Jeff}, \text{Jello}, \text{Sally}, \text{Sally}, \text{Molly}, \text{Jolly}\} = A \cup B \cup C.$$

$$S \setminus (A \cap B) = \{x \in S \mid x \notin A \cap B\}$$

$x \notin A \cap B$ means

"x is not in $A \cap B$"

not "x is in $A \cap B$" $\equiv \neg (x \in A \cap B)$

$$\equiv \neg (x \in A \text{ and } x \in B)$$

$$\equiv \neg (x \in A) \vee \neg (x \in B)$$

$$\equiv x \notin A \text{ or } x \notin B$$

$S \setminus (A \cap B) = \{x \in S \mid x \notin A \text{ or } x \notin B\}$ (5)

$= \{x \in S \mid x \notin A\} \cup \{x \in S \mid x \notin B\}$

$= S \setminus A \cup S \setminus B.$

$= \{$ Sally, Sally, Molly, Jolly $\} \cup$

$\{$ ~~June~~ June, ~~Sarah~~, Jill, Jeff, Sally, Sally, Molly, Jolly $\}$

$= \{$ Jill, Jeff, Sally, Sally, Molly, June, Jolly $\}$.