Eg: $\{a, b, c, d, e\}$

$P(5,2) = C(5,2) \cdot P(2,2)$

$\{a,b\}$ $\{a,c\}$ $\{a,d\}$ $\{a,e\}$ $\{b,c\}$ $\{b,d\}$ $\{b,e\}$

ab	ac	ad	ae	bc	bd	be
ba	ca	da	ea	cb	db	eb

$\{c,d\}$ $\{c,e\}$ $\{d,e\}$

cd	ce	de
dc	ec	ed

We have written down 20 of the 2-permutations ~~2-combinations~~

Know: $P(5,2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2} = 20$

So these are all the 2-permutations.

$$C(5,2) = \frac{P(5,2)}{P(2,2)} = \frac{\left(\frac{5!}{(5-2)!}\right)}{\left(\frac{2!}{(2-2)!}\right)} = \frac{\left(\frac{5!}{3!}\right)}{\left(\frac{2!}{0!}\right)} = \frac{20}{2} = 10$$

$$C(5,2) = \frac{5!}{(5-2)!\,2!}$$

Eg.: A bag contains

- 3 red marbles,
- 3 blue marbles,
- 3 green marbles,
- 2 yellow marbles.

11 marbles total. Assume all the marbles are distinguishable, say e.g. that the marbles all have numbers.

a) How many sets of four marbles are possible?

This is equivalent to asking how many 4-combinations are there in a set of 11 elements?

$$C(11,4) = \frac{11!}{(11-4)!\,4!} = \frac{11!}{7!\,4!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{7!}\,4!}$$

$$= \frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2} = 11\cdot 5\cdot 3\cdot 2$$

$$= 11\cdot 30$$

$$= 330.$$

$$\frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2} = \frac{11\cdot(5\cdot\cancel{2})\cdot(\cancel{3}\cdot 3)\cdot(\cancel{4}\cdot 2)}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 11\cdot 5\cdot 3\cdot 2$$

b) How many sets of four are there such that each one is a different color? ③

1) · Choose a red (3 ways)

2) · Choose a green (3 ways)

3) · Choose a blue (3 ways)

4) · Choose a yellow (2 ways)

Mult principle $\Rightarrow$ $3 \cdot 3 \cdot 3 \cdot 2 = 54$ ways to choose such a set.

c) How many sets of 4 such that at least 2 are red?

Case 1: 2 are red.

$C(3,2) = \frac{3!}{(3-2)!\,2!} = \frac{3 \cdot 2}{2} = 3$ ways to choose 2 reds.

$C(8,2) = \frac{8!}{(8-2)!\,2!} = \frac{8!}{6!\,2} = \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot 2} = \frac{56}{2} = 28$ ways to choose other 2

$3 \cdot 28 = 84$ ways to choose such a set.

Case 2: 3 are red.

1 way to choose the reds.

$$C(8,1) = \frac{8!}{(8-1)!\,1!} = \frac{8\cdot 7!}{7!} = 8.$$

8 such sets.

So there $84+8=92$ ways to ~~ad~~ choose a set of 4 marbles with at least 2 reds.

d) How many sets of 4 are there in which none are red, but at least one is green?

Case 1: 1 Green

3 ways to choose a green.

$$C(5,3) = \frac{5!}{(5-3)!\,3!} = \frac{5\cdot 4\cdot \cancel{3}\cdot \cancel{2}}{2\cdot \cancel{3}\cdot \cancel{2}} = \frac{20}{2} = 10$$

$3\cdot 10 = 30$ ways to choose a set of 4 w/no red, 1 green.

Case 2: 2 Green

$C(3,2) = 3$ ways to choose 2 greens

$$C(5,2) = \frac{5!}{(5-2)!\,2!} = \frac{5!}{3!\cdot 2!} = \frac{5\cdot 4\cdot \cancel{3!}}{\cancel{3!}\,2} = \frac{20}{2} = 10$$

30 ways to choose a set of 4 w/no red, 2 green.

Case 3: 3 greens

1 way to choose greens

$$C(5,1) = \frac{5!}{(5-1)!\,1!} = \frac{5!}{4!} = 5.$$

5 ways to choose a set of 4 w/3 green no red.

So there are $30 + 30 + 5 = 65$ ways to choose a set of 4 w/ at least one green and no reds.

Next time: Show you how to compute

$(x+y)^n$.

E.g.: $(x+y)^4 = x^4 + 4x^3y + \overset{6}{\cancel{8}}x^2y^2 + 4xy^3 + y^4$.