1/29/16 

E.g.: Suppose you're in an ice cream parlor

chocolate or vanilla ice cream

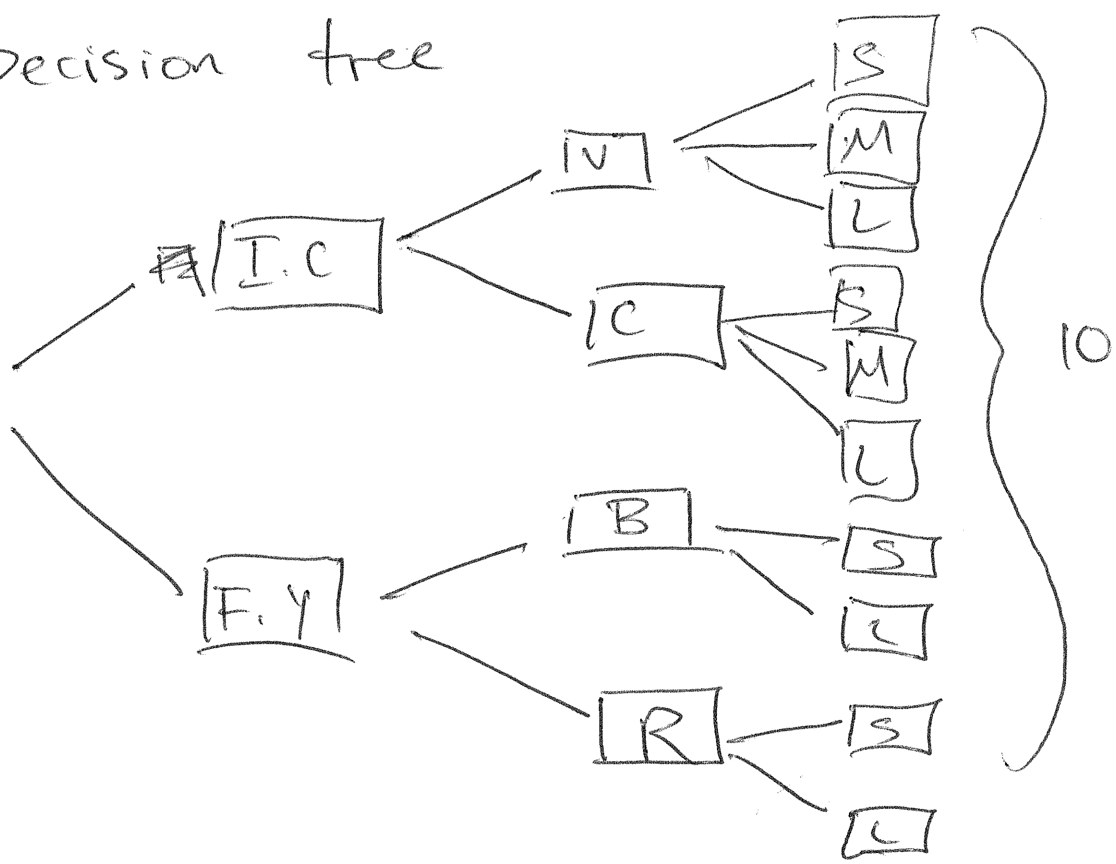
banana or raspberry frozen yogurt

3 cone sizes: small, med, large (ice cream)

2 cup sizes: small, large (frozen yogurt).

$3 \cdot 2 + 2 \cdot 2 = 6 + 4 = 10$ choices.

Decision tree



Eg: You have four letters, k, e, r, e. ②

Say you want to enumerate all possible four letter words with these four letters. How many are there?

$\_\ \_\ \_\ \_$

Think: 4 choices for the first letter
3 choices for the second letter
2 choices for the third letter
1 choice for the fourth letter.

Sequence of choices; use the multiplication principle, there are

$$4 \cdot 3 \cdot 2 \cdot 1 = 24.$$

possible four-letter words. But this is wrong.

Imagine you can distinguish the two e's. Say, for instance, as scrabble tiles one e has a crack in it, the other does not.

Choose r to be the first letter.
Choose e w/ crack to be second
Choose e w/o crack to be third
choose k last.

reek

Choose r to be the first letter,
Choose e w/o crack to be the second,
Choose e w/ crack to be the ~~k~~ third,
Choose k last.

reek

Two sequences of choices that are not the same, but have ~~a different outcome~~ the same outcome. This implies that the multiplication principle does not apply here.

The right way to count here is to observe that once a location has been chosen for r and k, the location of the e's is determined.

There are four possible locations for the

r. Once the location of the r is chosen, there are 3 locations for the k, hence there are

$$4 \cdot 3 \cdot 1 = 12$$

possible four letter words.

| | | | |
|---|---|---|---|
| r k e e | k r e e | k e r e | k e e r |
| r e k e | e r k e | e k r e | e k e r |
| r e e k | e r e k | e e r k | e e k r |

# Permutations & Combinations



Def$^n$: A permutation of a set with $n$ elements is an ordered list of these items.

E.g.: Say we have the four letters a, b, c, and d. ($\{a,b,c,d\}$)

How many ways can we permute these letters? Some permutations are:

| | | |
|---|---|---|
| a b c d | b a c d | c a b d |
| a c b d | b c a d | c b a d |
| a c d b | b c d a | c b d a |

etc.

| | | | | |
|---|---|---|---|---|
| There are four choices | for | the | first |
| three " | " | " | second |
| 2 " | " | " | third |
| 1 choice | " | " | fourth |

So there are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ permutations.



Say we have $n$ elements in the set $S$.

1st step • Choose an element, take it out of the set.

2nd step • Choose an element from the remaining elements. Take that out.

⋮

$(n-1)^{st}$ step • Choose one of the remaining two elements. Take it out.

$n^{th}$ step • Choose the last element.

There are $n$ ways to choose the first, $n-1$ ways to choose the second, ..., 2 ways to choose the $(n-1)^{st}$, 1 way to choose the $n^{th}$.

The # of permutations of a set with $n$ elements is

$$n! = n \cdot (n-1)(n-2) \cdots (2)(1)$$

where $n!$ is called "n factorial".

E.g.: By definition, $0! = 1$

$1! = 1$ $\qquad$ $3! = 3 \cdot 2 \cdot 1 = 6$

$2! = 2 \cdot 1$ $\qquad$ $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 120.$

$$\begin{array}{r} {}^{2}24 \\ \times\ 5 \\ \hline 0 \end{array}$$

E.g.: Compute $30!/29! = \dfrac{30 \cdot 29 \cdot 28 \cdot 27 \cdots 2 \cdot 1}{29 \cdot 28 \cdot 27 \cdots 2 \cdot 1} = 30$

$700!/699! = 700$

$$\frac{20!}{16!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16!}}{\cancel{16!}} = 20 \cdot 19 \cdot 18 \cdot 17$$