Eg:
$T = \mathbb{N} = \{\text{positive integers}\}$
$S = \{n \in \mathbb{N} \mid n \neq 1\}$
If we were to try using the formula from last time

$$|T \setminus S| = |T| - |S|$$
$$= \infty - \infty$$
$$= 0$$

but this is wrong. Can't do arithmetic with infinity.

$$|T \setminus S| = |\{n \in \mathbb{N} \mid n \notin S\}| = |\{1\}| = 1.$$

Eg: $T = \mathbb{Z}$, $S = \mathbb{N}$

$$|T \setminus S| = |\{n \in \mathbb{Z} \mid n \leq 0\}| = \infty.$$

6.3 Decision Algorithms

Addition Principle

Eg: At a restaurant you can choose from
- 8 chicken dishes
- 10 beef dishes

- 4 Seafood dishes
- 12 vegetarian dishes

How many are there to choose from?

$$8+10+12+4=34.$$

When choosing amongst r disjoint alternatives suppose that there are

- n_1 outcomes for the first alternative
- n_2 " " " second "
- ⋮
- n_r " " " r^{th} "

with no two of these outcomes the same, then there are

$$n_1+n_2+\cdots+n_r$$

possible outcomes.

Rmk: If O_1 as the set of outcomes for alternative 1, O_2 for alternative 2, …, O_r set of outcomes for alternative r, then the set of possible outcomes is

$$\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2 \cup \cdots \cup \mathcal{O}_r\,,$$

and the cardinality of $\mathcal{O}$ is the ~~set~~ number of all possible outcomes. Stipulating that no 2 outcomes are ~~of~~ the same says that these sets are all disjoint and

$$|\mathcal{O}| = |\mathcal{O}_1| + |\mathcal{O}_2| + \cdots + |\mathcal{O}_r|$$

$$= n_1 + n_2 + \cdots + n_r.$$

E.g.: $\mathcal{O}_1 = \{\text{chicken dishes}\}$ $\quad |\mathcal{O}_1| = 8$

$\mathcal{O}_2 = \{\text{beef dishes}\}$ $\quad |\mathcal{O}_2| = 10$

$\mathcal{O}_3 = \{\text{seafood dishes}\}$ $\quad |\mathcal{O}_3| = 4$

$\mathcal{O}_4 = \{\text{vegetarian dishes}\}$ $\quad |\mathcal{O}_4| = 12$

$$\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{O}_3 \cup \mathcal{O}_4$$

$$= \{\text{all dishes at the restaurant}\}.$$

$$\mathcal{O}_1 \cap \mathcal{O}_2 = \emptyset \qquad \mathcal{O}_1 \cap \mathcal{O}_3 = \emptyset$$

$$\ldots \quad \mathcal{O}_i \cap \mathcal{O}_j = \emptyset \quad i \neq j$$

$$|\mathcal{O}| = |\mathcal{O}_1| + |\mathcal{O}_2| + |\mathcal{O}_3| + |\mathcal{O}_4| = 8 + 10 + 4 + 12 = 34.$$

Say $O_5 = \{\text{Pasta category}\}$, $|O_5| = 1$

this dish has chicken & pasta,

say its listed on the menu twice – once

under chicken dishes, once under pasta,

then $O_5 \cap O_1 \neq \emptyset$, $|O_5 \cap O_1| = 1$.

$$|O| = |O_1| + |O_2| + |O_3| + |O_4| + |O_5| - |O_1 \cap O_5|.$$

$$= 8 + 10 + 4 + 12 + 1 - 1$$

$$= 34.$$

Multiplication Principle

Eg.: A restaurant has 5 appetizers, 8 entrées, and 3 desserts. If each meal has one of each, how many possible meals?

There are $5 \cdot 8 \cdot 3 = 120$.

Think about a meal as an ordered triple of an appetizer, an entrée, and a desert.

$A = \{\text{appetizers}\} = \{a_1, a_2, a_3, a_4, a_5\}$

$E = \{\text{entrees}\} = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

$D = \{\text{desserts}\} = \{d_1, d_2, d_3\}$

$(a_1, e_1, d_1) \quad (a_2, e_1, d_1) \quad \cdots$

$(a_1, e_2, d_2) \quad (a_2, e_2, d_2) \quad \cdots$

$\vdots \qquad\qquad \vdots$

At any step, choosing something different from any 3 steps gives a distinct outcome.

The set of all possible meals is the Cartesian product of those sets

$$M = A \times E \times D$$

$$|M| = |A| \cdot |E| \cdot |D|.$$

Multiplication Principle

When making a sequence of choices with r steps, suppose that

step 1 has n_1 possible outcomes,

" 2 " n_2 " " ,

$\vdots$

" r " n_r " "

and ~~ea~~ each sequence of choices results in a distinct outcome. Then there are

$n_1 \cdot n_2 \cdot \ldots \cdot n_r$ possible outcomes.

Rmk: $\mathcal{O} = \mathcal{O}_1 \times \mathcal{O}_2 \times \cdots \times \mathcal{O}_r$

$$|\mathcal{O}| = |\mathcal{O}_1||\mathcal{O}_2| \cdots |\mathcal{O}_r|.$$

E.g.: An ice cream shop has
15 flavors of ice cream,
5 flavors of frozen yogurt,
3 cone sizes for ice cream,
2 cup sizes for frozen yogurt.

How many choices for a dessert?

desserts = # ways to get ice cream
+
ways to get frozen yogurt

ways to get ice cream = (# flavors)(# cone sizes)
$= 15 \cdot 3 = 45.$

ways to get frozen yogurt = (# flavors)(# cup sizes)
$= (5)(2) = 10$

desserts $= 45 + 10 = 55.$