6: Sets & Counting 1/22/16

Def^n: A <u>set</u> ~~on~~ is an unordered collection of objects, referred to as elements or members of the set.

<u>E.g.:</u> $S = \{1, 2, 3, 4\}$

set

the elements of S are $1, 2, 3,$ and 4.

<u>E.g.:</u> $S = \{\text{cow}, \text{dog}, \text{horse}\}$.

<u>Rmk</u>: Sets do <u>not</u> capture repitition.

The sets $S = \{1, 2, 3, 4\}$ and $T = \{1, 2, 2, 3, 4\}$ are the same.

Def^n: A set S is a <u>subset</u> of the set T if every element of S is also an element of T. Venn Diagram: 

A subset S of T is said to be a <u>proper</u> subset of T if S and T are not the same.

E.g.: $S = \{1,2,3\}$, $T = \{1,2,3,4,5,6\}$

We denote "S is a subset of T" by

$$S \subseteq T \quad (\text{think } \leq)$$

We denote "S is a proper subset of T" by

$$S \subset T. \quad (\text{think } <)$$

Rmk: We say two sets S and T are equal if $S \subseteq T$ and $T \subseteq S$; we write $S = T$.

if and only if

Notation: If x is an element of S we write $x \in S$ to denote this relationship. Formally read "x is an element of S", commonly read "x is in S."

Defn: If a set S has finitely many elements (i.e. you ~~can~~ could write down all of them), we say S is a finite set.

If S is not finite, we say S is infinite.

Common Infinite Sets

$\mathbb{N} = \{\text{all positive integers}\}$ ← "natural numbers"

$\cap|$

$\mathbb{Z} = \{\text{all } \cancel{\text{positive}} \text{ integers}\}$

$\cap|$

$\mathbb{Q} = \{\text{all rational numbers}\}$

$\cap|$

$\mathbb{R} = \{\text{all real numbers}\}$

$\cup|$

$\mathbb{C} = \{\text{all complex numbers}\}$.

"Set Builder Notation"

condition

$\{\text{all even, positive integers}\} = \{x \in \mathbb{N} \mid 2 \text{ divides } x\}$

"such that"

$\mathbb{C} = \{x + iy \mid x \in \mathbb{R}, y \in \mathbb{R}\}$, $i = \sqrt{-1}$

$\mathbb{Q} = \{\frac{x}{y} \mid x \in \mathbb{Z}, y \in \mathbb{Z}\}$

$\{\text{first 5 million positive integers}\} = \{x \in \mathbb{N} \mid x \leq 5000000\}$

Def^{n}: The <u>cardinality</u> of a set S, denoted $|S|$ (by me) or $n(S)$ (by the book) or $\# S$ (by others), is the number of elements in S if S is finite, infinity otherwise.

E.g.: $S = \{\text{cow, dog, moose}\}$

$|S| = \cancel{8}.\ 3.$

$|\mathbb{N}| = \infty$

$|\mathbb{R}| = \infty$

Set Operations

Def^{n} The set with no elements is the empty set, $\emptyset$.

~~Let~~ Let A and B be sets.

① The union of A and B, written $A \cup B$, is the set of elements either in A or in B. (or ~~&~~ in both)

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$ (think disjunction/or)

② The intersection of A and B, written $A \cap B$, is the set of common elements

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$ (think conjunction/and)

③ If $A \subseteq B$, the complement of A in B, written

$B\setminus A$ (me), A' (book), $B-A$ (others)

is the set of elements in B but not in A,

$$B\setminus A = \{x\in B \mid x \notin A\}$$ (think negation)

"not in"

Eg: $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$.

$$\mathbb{Z}\setminus\mathbb{N} = \{\text{negative integers}\} \cup \{0\}$$

$$\mathbb{Q}\setminus\mathbb{N} = \{\text{negative integers}\} \cup \left\{\frac{a}{b} \,\middle|\, b\neq 1, a\in\mathbb{Z}\right\}.$$