Modus Tollens / Indirect Reasoning

Let p and q be propositions. If the implication $p \Rightarrow q$ is true, but the consequence q is false, then p must also be false.

E.g.: If b^2-4ac is negative, then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

are both not real numbers.

Assume that $ax^2+bx+c=0$ has 2 real solutions. From ~~the~~ this we infer that

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

are both real numbers. Therefore b^2-4ac is not negative.

Disjunctive Syllogism (One or the other)

Let p and q be propositions. If one of p or q is true, but one is known to be false, then the other is true.

E.g.: Either the cook did it or the butler did it. The butler did not do it. Therefore the cook did it.

Formally, this is one of two implications

$$[(p \overset{\text{or}}{\vee} q) \overset{\text{and}}{\wedge} \underset{\text{not}}{\neg} q] \Rightarrow p, \quad \text{or}$$

$$[(p \vee q) \wedge \neg p] \Rightarrow q.$$

Formally, Modus Tollens is the implication

$$[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$$

p	q	$p \Rightarrow q$	$\neg q$	$(p \Rightarrow q) \wedge \neg q$	$\neg p$	$[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Recall: The contrapositive of the implication $p \Rightarrow q$ is $\neg q \Rightarrow \neg p$. Those two implications are logically equivalent.

p	q	$p \Rightarrow q$	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

colums are the same

Short-hand: $p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

"logically equivalent to"

De Morgan's Laws

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Loose analogue: $-(a+b) = -a + -b$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

On page A10, there is a small list of useful logical equivalences.