1/15/16 ①

Rmk: The book/webAssign uses

① $\sim$ to mean $\neg$
(i.e. $\sim p$ and $\neg p$ mean the same thing).

② $\rightarrow$ to mean $\Rightarrow$ ($\leftrightarrow$ means $\Leftrightarrow$).
(i.e. $p \rightarrow q$ and $p \Rightarrow q$ mean the same thing).

Def^n: An <u>argument</u> is a list of <u>premises</u> (or hypothesis), $P_1, P_2, \cdots, P_n$, followed by a proposition, C, called the <u>conclusion</u>.

A <u>valid argument</u> satisfies

$$P_1 \wedge P_2 \wedge \cdots \wedge P_n \Rightarrow C$$

is a tautology. (An argument is invalid if not valid)

We say an argument is <u>sound</u> if it is valid and all the premises are true.

Def^n (Modus Ponens / Conditional Elimination / Direct Reasoning)

Let p and q be propositions. If the implication $p \Rightarrow q$ is true, and p is true, then q is true.

Truth Table

$P_1 : p \Rightarrow q$

$P_2 : p$

$C : q$

Argument (Modus Ponens)

$$((p \Rightarrow q) \wedge p) \Rightarrow q$$

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge p$	$[(p \Rightarrow q) \wedge p] \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

E.g.: If $x^2 = 4$, then $x = 2$.

hypothesis: $x^2 = 4$

conclusion: $x = 2$.

If $x = -2$, this implication is FALSE.

If $x = 2$, this implication is true.

Common Algebra Mistake: Solve $x^2 = 4$.

$2^2 = 4$, therefore $x = 2$

But you forgot $x = -2$.

E.g.: Let a, b, c be integers, $a \neq 0$.

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(Quadratic Formula).}$$

Pf:

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$ax^2 + bx + c = 0$$

$$\Rightarrow ax^2 + bx = -c$$

$$\Rightarrow x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\Rightarrow x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad \blacksquare$$

Q.E.D. symbol

(Quod ~~Erat~~ Erat Demonstratum)

Prop: $1=2$

<u>Pf</u>: Let $a=b$.

$\Rightarrow a \cdot a = a^2 = a \cdot b$ (mult both sides by a).

$\Rightarrow a^2 + a^2 = a^2 + ab$ (add a^2 to both sides)

$\Rightarrow 2a^2 = a^2 + ab$ (rewrite)

$\Rightarrow 2a^2 - 2ab = a^2 + ab - 2ab = a^2 - ab$ (sub $2ab$ both sides)

$\Rightarrow 2(a^2 - ab) = a^2 - ab$ (factor out 2 on left)

$\Rightarrow 2 = 1$ (divide both sides by $a^2 - ab$). ∎

Not sound. Divided by zero in the second to last statement.

<u>E.g.</u>: a, b, c integers, $a \neq 0$.

$ax^2 + bx + c = 0$ has 2 real solutions if and only if $b^2 - 4ac > 0$.

<u>Recall</u>: Complex numbers are of the form $x + iy$, where x, y are real numbers, $i = \sqrt{-1}$.

Pf: Assume ax^2+bx+c has 2 real solutions. 5

They are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\Rightarrow$ b^2-4ac must be non-negative.

If $b^2-4ac=0$, then

$$x = \frac{-b \pm \sqrt{0}}{2a} = \frac{-b}{2a}$$

only one solution, so b^2-4ac must be positive.

Assume $b^2-4ac>0$. Then the solutions are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\Rightarrow$ 2 solutions are

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \& \quad x = \frac{-b-\sqrt{b^2-4ac}}{2a}. \blacksquare$$