# Conditionals / Implications

1/13/16 ①

Def$^n$: Let $p$ and $q$ be propositions. The implication ~~"q ... q"~~ "If $p$, then $q$", denoted $p \Rightarrow q$, is defined by the truth table

| | $p$ | $q$ | $p \Rightarrow q$ |
|---|---|---|---|
| ① | T | T | T |
| ② | T | F | F |
| ③ | F | T | T |
| ④ | F | F | T |

The proposition $p$ is called the hypothesis (or antecedent or premise) and the proposition $q$ is called the conclusion (or consequence).

Rmk: In the latter two entries of the truth table, the implication $(p \Rightarrow q)$ is ~~called~~ said to be vacuously true.

E.g.: "If you get a 100 on the Final Exam, then you will get an A in the class."

$p$: "You got a 100 on the final exam."

$q$: "You got an A in the class"

①



- Assume you got a 100 ~~in the~~ on the Final Exam and you get an A in the class
  - This makes you happy.

② • Assume you get a 100 on the Final Exam, but you don't get an A in the class.
  - You feel cheated.

③ • Assume you do not get a 100 on the Final Exam, but You get an A in the ~~class~~. class.
  - You feel happy.

④ • Assume you do not get a 100 on the final exam, ~~b~~ and you do not get an A in the class.
  - Not necessarily happy, but not cheated.

E.g.: "If it Sunny tomorrow, then we will go to the beach." - Common

E.g.: "If today is Friday, then $2+4=7$."
- Uncommon, but valid.

True every day except on Friday.

Very false on Friday. $(T \Rightarrow F)$.

# Converse and Contrapositive

Def$^n$: Let $p$ and $q$ be propositions. The converse of $p \Rightarrow q$ is the implication $q \Rightarrow p$.

The contrapositive of $p \Rightarrow q$ is the implication $\neg q \Rightarrow \neg p$.

E.g.: The converse of

"If today is Friday, the $2+4=7$" *

is

"If $2+4=7$, then today is Friday."

Since $2+4=7$ is always false, the latter implication is always true.

The contrapositive of * is

"If $2+4 \neq 7$, then it is not Friday."

Def$^n$: Let $p$ and $q$ be propositions. The biconditional "$p$ if and only if $q$," denoted $p \Leftrightarrow q$, is the proposition that is true whenever both $p \Rightarrow q$ is true and $q \Rightarrow p$ is true.

# Truth Table

| $p$ | $q$ | $p \Rightarrow q$ | $q \Rightarrow p$ | $p \Leftrightarrow q$ | $(p \Rightarrow q) \wedge (q \Rightarrow p)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

Obs: $p \Leftrightarrow q$ is true only when $p$ and $q$ have the same truth value

Def^n: A proposition that is always true is called a tautology and a proposition that is always false is called a contradiction.

Def^n: We say two propositions, $p$ and $q$, are logically equivalent if $p \Leftrightarrow q$ is a tautology. This means that $p$ and $q$ always have the same truth value.