Logic 1/11/16

Def$^{\underline{n}}$: A proposition is a declarative statement that is either true or false. The truth values are denoted by T and F.

definition

E.g.: ① Washington D.C. is the capital of the united states. (T)

② $2+2=4$ (T)

③ $3+4=8$ (F)

E.g. (Non-Examples): ① Read this carefully.

② This statement is false.

If this statement is false, then it is true (?)

If this statement is true, then it's false (?!)

We declare this is not a proposition.

③ (Liar's Paradox): This statement is a lie.

Same as above.

~~Defn~~

Rmk: We often give propositions lower letters for names, much like variables in Algebra. Usually something like p & q.

Remark

Def^{n}: Let p be a proposition. The statement

"it is not the case that p"

is called the <u>negation</u> of p, denoted by $\neg p$, and is usually read "not p".

The truth value of $\neg p$ is opposite that of p.

E.g.: p: "Today is Monday" (T)

$\neg p$: "Today is not Monday" (F)

Truth Tables

Def^{n}: A <u>truth table</u> displays the relationships between the truth values of propositions.

E.g.:

p	$\neg p$
T	F
F	T

Connectives

Def^{n}: Let p and q be propositions. The <u>conjunction</u> <u>of p and q</u>, denoted $p \wedge q$, is the proposition "p and q". It is true when both p and q are true and false otherwise.

Eg.: ① p: "Today is Monday." (T)

q: "2+2=4" (T)

$p \wedge q$: "Today is Monday and 2+2=4." (T)

② p: "Today is Tuesday." (F)

q: "Today is Monday." (T)

$p \wedge q$: "Today is Monday and Tuesday." (F)

The truth table for conjunction is

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Defn: Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "p or q," which is true whenever p and q are not both false, false otherwise.

Eg: ① p: "Today is Monday." (T)

q: "2+2=5." (F)

$p \vee q$: "Today is Monday or $2+2=5$." (T)

2) p: "$2+2=5$" (F)

q: "Today is Tuesday." (F)

$p \vee q$: "$2+2=5$ or Today is Tuesday." (F)

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F