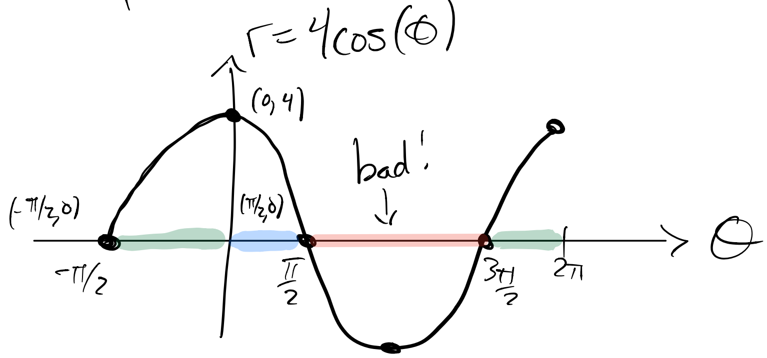
4/26/18

E.g: $r^2 = 4\cos(\theta)$

Graph on the Cartesian plane



r is a real number, so $r^2 < 0$ has no solutions.

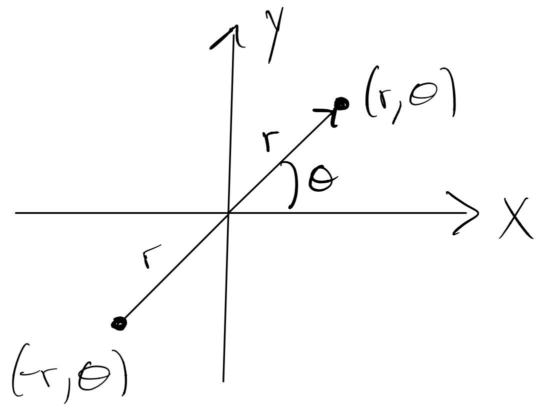
Instead of looking at θ in $[0, \pi/2] \cup [3\pi/2, 2\pi]$, look at θ in $[-\pi/2, \pi/2]$

For a given value of θ, r has two choices:

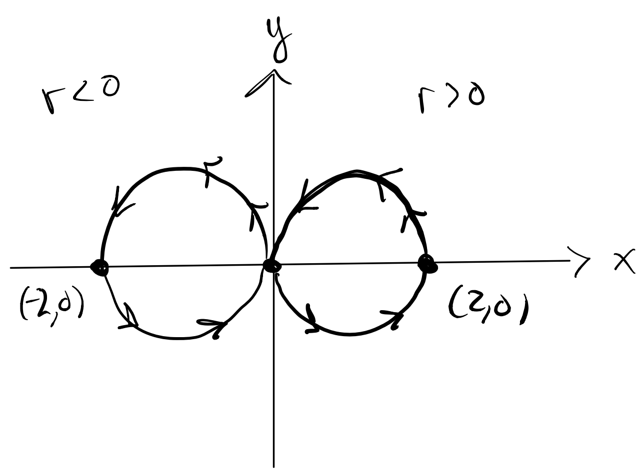
$$r = \sqrt{4\cos(\theta)} = 2\sqrt{\cos(\theta)} \quad \text{or}$$

$$r = -\sqrt{4\cos(\theta)} = -2\sqrt{\cos(\theta)}.$$

This says we have



Handle $r > 0$ case first.



Andrew Kustin, Teaching, Math 142
Old Exams/Solutions.

Substitution – Modifies the interval

Term-by-Term Int/Diff
Multiplaction/Addition $\Big\}$ Don't change where the series converges to the function

E.g: Find the Maclaurin series for

$$\frac{1}{1+x^2}$$

Tell me where it converges to the function.

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad (-1,1)$$

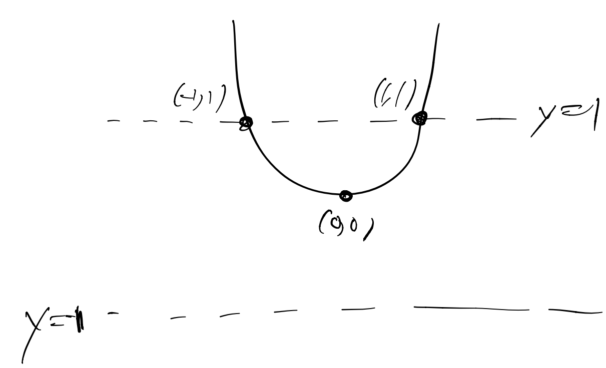
$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{on } (-1,1), \ |x|<1$$

Need $-1 < -x^2 < 1$

$1 > x^2 > -1$

$-1 < x^2 < 1$

So $-1 < x < 1$



E.g.: $$\frac{1}{1-2x} = \sum_{n=0}^{\infty} (2x)^n = \sum_{n=0}^{\infty} 2^n x^n$$

provided

$$-1 < 2x < 1$$

$$\Updownarrow$$

$$-\frac{1}{2} < x < \frac{1}{2}.$$

Should know

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \leftarrow \text{workhorse}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n} \quad \text{or} \quad \sum_{n=0}^{\infty} \int (-1)^n x^n dx$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$$