Activity 1

1. Given the Cartesian equation $y = 5(x-2)$, find two different parameterizations of the curve.

   Possible Solutions: Using the natural parameterizations, we let $x(t) = t$, then $y(t) = 5(t-2)$ and take the parameter interval $-\infty < t < \infty$.

   Alternatively, we can use $x(t) = t+2$, then you get $y(t) = 5(t+2-2) = 5t$ and take the parameter interval $-\infty < t < \infty$.

2. A hiker in Yellowstone National Park is traveling along a path defined by the parametric equations $x(t) = 80 - 4t$, $y(t) = 3t$. A bear leaves another area of the park and walks along the path described by $x(t) = 2t$, $y(t) = 20 + t$.

   (a) Consider the Cartesian equations for the hiker and bear. Do these suggest that the paths of the hiker and bear intersect?

   (b) If the paths meet, do the hiker and bear reach the intersection at the same time t?

   Possible Solutions: To determine whether the paths intersect, find the Cartesian representations of the given curves:

   Hiker: Solve for t in terms of x, $x = 80 - 4t \Rightarrow t = 20 - \frac{x}{4}$, which we then plug in for t in the equation for y to get $y = 3t = 3(20 - \frac{x}{4}) = \frac{-3}{4}x + 60$.

   Bear: Solve for t in terms of x, $x = 2t \Rightarrow t = \frac{x}{2}$, and then plug this into the equation for y to get $y = 20 + t = \frac{1}{2}x + 20$.

   So we have two lines in the plane, one with slope $\frac{-1}{4}$ and the other with slope $\frac{1}{2}$, so they are not parallel and so they must intersect somewhere.

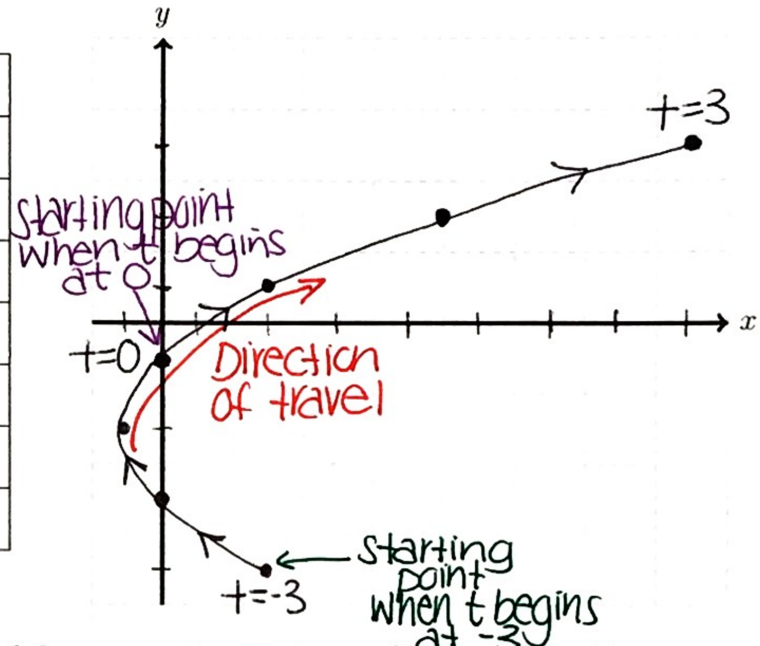
   Now find out whether the bear and hiker collide, we just check if the x and y coordinates are equal at the same t, so we set the $x(t)$'s equal and solve for t and do the same for the $y(t)$'s and check if the t's are the same:

   $80 - 4t = 2t \Rightarrow t = 13.\overline{3}, 20 + t = 3t \Rightarrow t = 10$. Since these two t values are not the same, we conclude that the bear and the hiker do not meet!

Handout - Graphing Parametric Equations

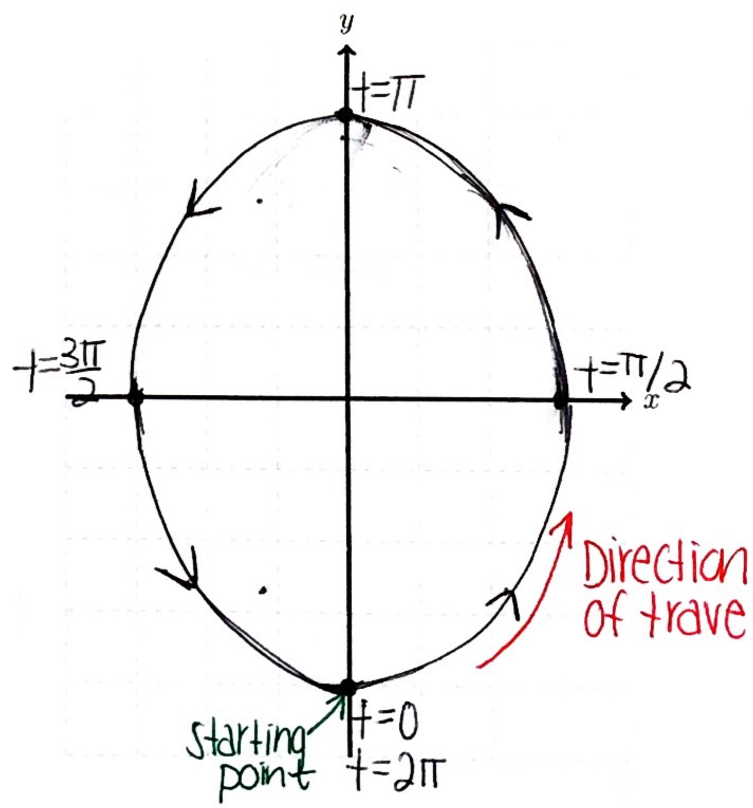
Consider how to sketch the graph of the parametric equations: $x(t) = t^2 + 2t$, $y(t) = 2t - 1$ for $\cancel{0}$ -3 $\leq t \leq 3$. Or for $\cancel{-3}$ 0 $\leq t \leq 3$.

t	x(t)	y(t)
-3	3	-7
-2	0	-5
-1	-1	-3
0	0	-1
1	3	1
2	8	3
3	15	5



Now consider how to sketch the graph of the parametric equations: $x(t) = 3\sin(t)$, $y(t) = -4\cos(t)$ for $0 \leq t \leq 4\pi$.

t	x(t)	y(t)
0	0	-4
$\frac{\pi}{4}$	$\frac{3\sqrt{2}}{2} \approx 2.12$	$-2\sqrt{2} \approx -2.828$
$\frac{\pi}{2}$	3	0
$\frac{3\pi}{4}$	$\frac{3\sqrt{2}}{2}$	$2\sqrt{2}$
π	0	4
$\frac{5\pi}{4}$	$-\frac{3\sqrt{2}}{2}$	$2\sqrt{2}$
$\frac{3\pi}{2}$	-3	0
$\frac{7\pi}{4}$	$-\frac{3\sqrt{2}}{2}$	$-2\sqrt{2} \approx -2.828$
2π	0	$\cancel{-2\sqrt{2} \approx -2.83}$ -4

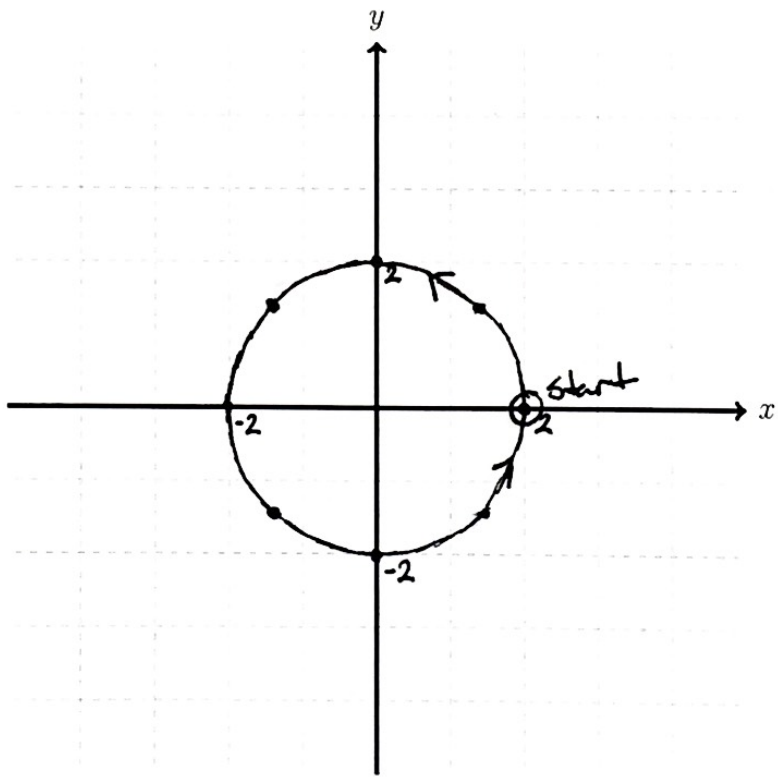


Activity 2

1. Sketch the graph of the parametric equations: $x(t) = 2\cos(t)$, $y(t) = 2\sin(t)$ for $0 \le t \le 2\pi$. Indicate the start point and direction of travel.

t	x(t)	y(t)
0	2	0
$\frac{\pi}{4}$	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{2}$	0	2
$\frac{3\pi}{4}$	$-\sqrt{2}$	$\sqrt{2}$
π	-2	0
$\frac{5\pi}{4}$	$-\sqrt{2}$	$-\sqrt{2}$

$\sqrt{2} \approx 1.414$

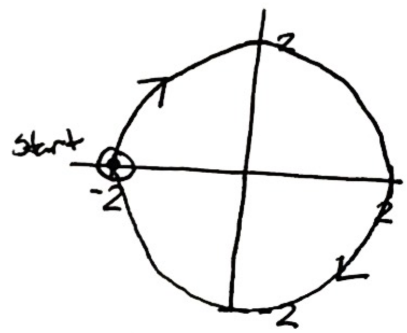


2. Describe in words what happens to the previous graph if we change the values of t to $-\pi \leq t \leq \pi$. What about $0 \leq t \leq 4\pi$?

First, if we considered $-\pi \leq t \leq \pi$ our start point would change from being $(2,0)$ to $(-2,0)$ and we would still travel counterclockwise through the circle.
Second, if we considered $0 \leq t \leq 4\pi$ our start point would stay at $(2,0)$ and we would still travel counterclockwise, but we would trace the circle twice.

3. Consider the parametric equations $x(t) = -2\cos(t)$, $y(t) = 2\sin(t)$ for $0 \leq t \leq 2\pi$. How is this curve different from the graph you sketched in Exercise 1?

t	x	y
0	-2	0
$\frac{\pi}{2}$	0	2
π	2	0
$\frac{3\pi}{2}$	0	-2
2π	-2	0



First we notice that $x(t)$ is negative compared to the $x(t)$ in Ex. 1.

From plugging in $t=0$ we see that we start at $(-2,0)$ instead of $(2,0)$.

From graphing the 1st couple of t values we notice that this graph travels clockwise instead of counterclockwise.