



MATH 122

FARMAN

6.1/6.2:
ANTIDERIVATIVES

POLYNOMIALS
EXPONENTIALS

THE INDEFINITE
INTEGRAL

SOME EXAMPLES

NEGATIVE
EXPONENTS

MATH 122

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Calculus for Business Administration and Social
Sciences



OUTLINE

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- Polynomials
- Exponentials
- The Indefinite Integral
- Some Examples
- Negative Exponents



ANTIDERIVATIVES

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DEFINITION 1

Given a function, f , we say an *antiderivative* of f is a function F such that

$$F'(x) = f(x).$$



REMARKS

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REMARK 1

- 1 By the Fundamental Theorem of Calculus, given an antiderivative, F , of f ,

$$\int_a^b f(x) dx =$$



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$$\int_a^b f(x) dx = \int_a^b F'(x) dx$$



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REMARK 1

- ① By the Fundamental Theorem of Calculus, given an antiderivative, F , of f ,

$$\int_a^b f(x) dx = \int_a^b F'(x) dx = F(b) - F(a).$$



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REMARK 1

- 1 By the Fundamental Theorem of Calculus, given an antiderivative, F , of f ,

$$\int_a^b f(x) dx = \int_a^b F'(x) dx = F(b) - F(a).$$

- 2 If f admits an antiderivative, F , then for any $c \in \mathbb{R}$, $F(x) + c$ is also an antiderivative because

$$\frac{d}{dx} (F(x) + c) =$$



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- ❷ If f admits an antiderivative, F , then for any $c \in \mathbb{R}$, $F(x) + c$ is also an antiderivative because

$$\frac{d}{dx} (F(x) + c) = \frac{d}{dx} F(x) + \frac{d}{dx} (c)$$



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$$\frac{d}{dx}(F(x) + c) = \frac{d}{dx}F(x) + \frac{d}{dx}(c) = f(x) + 0$$



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- 2 If f admits an antiderivative, F , then for any $c \in \mathbb{R}$, $F(x) + c$ is also an antiderivative because

$$\frac{d}{dx}(F(x) + c) = \frac{d}{dx}F(x) + \frac{d}{dx}(c) = f(x) + 0 = f(x).$$



MONOMIALS

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For any monomial $f(x) = ax^n$, $0 \leq n$, an antiderivative of f is

$$F(x) =$$



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For any monomial $f(x) = ax^n$, $0 \leq n$, an antiderivative of f is

$$F(x) = \frac{a}{n+1}x^{n+1} + c, c \in \mathbb{R}$$



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since

$$F'(x) =$$



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$$F'(x) = \frac{a}{n+1}(n+1)x^n$$



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For any monomial $f(x) = ax^n$, $0 \leq n$, an antiderivative of f is

$$F(x) = \frac{a}{n+1}x^{n+1} + c, \quad c \in \mathbb{R}$$

since

$$F'(x) = \frac{a}{n+1}(n+1)x^n = ax^n.$$



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Since we know the antiderivative for a monomial, given a polynomial

$$f(x) = \sum_{i=0}^n a_{n-i} x^{n-i}$$

we have the antiderivative

$$F(x) = \sum_{i=0}^n a_{n-i} \frac{1}{n-i+1} x^{n-i+1} + c, \quad c \in \mathbb{R}.$$



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This follows from



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This follows from

$$F'(x) = \frac{d}{dx} \left(\sum_{i=0}^n a_{n-i} \frac{1}{n-i+1} x^{n-i+1} + c \right)$$



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This follows from

$$\begin{aligned} F'(x) &= \frac{d}{dx} \left(\sum_{i=0}^n a_{n-i} \frac{1}{n-i+1} x^{n-i+1} + c \right) \\ &= \sum_{i=0}^n \frac{d}{dx} \left(a_{n-i} \frac{1}{n-i+1} x^{n-i+1} \right) \end{aligned}$$



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This follows from

$$\begin{aligned} F'(x) &= \frac{d}{dx} \left(\sum_{i=0}^n a_{n-i} \frac{1}{n-i+1} x^{n-i+1} + c \right) \\ &= \sum_{i=0}^n \frac{d}{dx} \left(a_{n-i} \frac{1}{n-i+1} x^{n-i+1} \right) \\ &= \sum_{i=0}^n a_{n-i} \frac{1}{n-i+1} (n-i+1) x^{n-i} \end{aligned}$$



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This follows from

$$\begin{aligned} F'(x) &= \frac{d}{dx} \left(\sum_{i=0}^n a_{n-i} \frac{1}{n-i+1} x^{n-i+1} + c \right) \\ &= \sum_{i=0}^n \frac{d}{dx} \left(a_{n-i} \frac{1}{n-i+1} x^{n-i+1} \right) \\ &= \sum_{i=0}^n a_{n-i} \frac{1}{n-i+1} (n-i+1) x^{n-i} \\ &= \sum_{i=0}^n a_{n-i} x^{n-i} \\ &= f(x). \end{aligned}$$



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Let $f(x) = 3x^2 + 2x + 5$.



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Let $f(x) = 3x^2 + 2x + 5$. Then for any $c \in \mathbb{R}$, an antiderivative of f is

$$F(x) =$$



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Let $f(x) = 3x^2 + 2x + 5$. Then for any $c \in \mathbb{R}$, an antiderivative of f is

$$F(x) = 3\frac{1}{3}x^3 + 2\frac{1}{2}x^2 + 5x + c$$



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$$F(x) = 3\frac{1}{3}x^3 + 2\frac{1}{2}x^2 + 5x + c = x^3 + x^2 + 5x + c.$$



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$$F(x) = 3\frac{1}{3}x^3 + 2\frac{1}{2}x^2 + 5x + c = x^3 + x^2 + 5x + c.$$

We can always check our solution:

$$F'(x) =$$



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$$F(x) = 3\frac{1}{3}x^3 + 2\frac{1}{2}x^2 + 5x + c = x^3 + x^2 + 5x + c.$$

We can always check our solution:

$$F'(x) = \frac{d}{dx}x^3 + \frac{d}{dx}x^2 + 5\frac{d}{dx}x$$



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We can always check our solution:

$$F'(x) = \frac{d}{dx}x^3 + \frac{d}{dx}x^2 + 5\frac{d}{dx}x = 3x^2 + 2x + 5$$



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We can always check our solution:

$$F'(x) = \frac{d}{dx}x^3 + \frac{d}{dx}x^2 + 5\frac{d}{dx}x = 3x^2 + 2x + 5 = f(x).$$



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Let $P(x) = P_0 e^{kx}$. Since $\frac{d}{dx} e^{kx} = k e^{kx}$, we observe that



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Let $P(x) = P_0 e^{kx}$. Since $\frac{d}{dx} e^{kx} = k e^{kx}$, we observe that

$$\frac{d}{dx} \frac{P_0}{k} e^{kx} =$$



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$$\frac{d}{dx} \frac{P_0}{k} e^{kx} = \frac{P_0}{k} \frac{d}{dx} e^{kx}$$



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Let $P(x) = P_0 e^{kx}$. Since $\frac{d}{dx} e^{kx} = k e^{kx}$, we observe that

$$\frac{d}{dx} \frac{P_0}{k} e^{kx} = \frac{P_0}{k} \frac{d}{dx} e^{kx} = \frac{P_0}{k} \cdot k \cdot e^{kx}$$



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$$\frac{d}{dx} \frac{P_0}{k} e^{kx} = \frac{P_0}{k} \frac{d}{dx} e^{kx} = \frac{P_0}{k} \cdot k \cdot e^{kx} = P_0 e^{kx} = P(x).$$

This implies

$$\frac{P_0}{k} e^{kx} + c, \quad c \in \mathbb{R}$$

is an antiderivative of $P(x)$.



EXPONENTS WITH ARBITRARY BASE

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If we want to integrate $P(x) = P_0 a^x$, then we can rewrite as

$$P(x) = P_0 a^x = P_0 e^{\ln(a)x}$$

so that



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$$P(x) = P_0 a^x = P_0 e^{\ln(a)x}$$

so that

$$\frac{d}{dx} \frac{P_0}{\ln(a)} a^x =$$



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so that

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$$P(x) = P_0 a^x = P_0 e^{\ln(a)x}$$

so that

$$\begin{aligned} \frac{d}{dx} \frac{P_0}{\ln(a)} a^x &= \frac{P_0}{\ln(a)} \frac{d}{dx} e^{\ln(a)x} \\ &= \frac{P_0}{\ln(a)} \left(\ln(a) e^{\ln(a)x} \right) \end{aligned}$$



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so that

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so that

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Therefore

$$\frac{P_0}{\ln(a)} a^x + c, \quad c \in \mathbb{R}$$

is an antiderivative of $P(x)$.



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DEFINITION 2

If f is a function with an antiderivative F , then the *indefinite integral* is the family of functions

$$\int f(x) dx = F(x) + c$$

where c is a constant.



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DEFINITION 2

If f is a function with an antiderivative F , then the *indefinite integral* is the family of functions

$$\int f(x) dx = F(x) + c$$

where c is a constant.

REMARK 2

Note that this immediately implies

$$\frac{d}{dx} \int f(x) dx = f(x).$$



PROPERTIES OF THE INDEFINITE INTEGRAL

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Assume that $\int f(x) dx$ and $\int g(x) dx$ exist. Then



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Assume that $\int f(x) dx$ and $\int g(x) dx$ exist. Then

- 1 The indefinite integral of a sum is the sum of the indefinite integrals:

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx.$$



PROPERTIES OF THE INDEFINITE INTEGRAL

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Assume that $\int f(x) dx$ and $\int g(x) dx$ exist. Then

- 1 The indefinite integral of a sum is the sum of the indefinite integrals:

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx.$$

- 2 Constants pass through the indefinite integral:

$$\int af(x) dx = a \int f(x) dx, a \in \mathbb{R}.$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int x^5 dx =$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int x^5 dx = \frac{1}{6}x^6 + c.$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int t^8 dt =$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int t^8 dt = \frac{1}{9}t^9 + c.$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int 12x^3 dx =$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int 12x^3 dx = 12 \int x^3 dx$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\begin{aligned}\int 12x^3 dx &= 12 \int x^3 dx \\ &= 12 \frac{1}{4} x^4 + c\end{aligned}$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\begin{aligned}\int 12x^3 dx &= 12 \int x^3 dx \\ &= 12 \frac{1}{4} x^4 + c \\ &= 3x^4 + c.\end{aligned}$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int q^3 - 6q^2 dq =$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\int q^3 - 6q^2 dq = \int q^3 dq - \int 6q^2 dq$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\begin{aligned}\int q^3 - 6q^2 dq &= \int q^3 dq - \int 6q^2 dq \\ &= \int q^3 dq - 6 \int q^2 dq\end{aligned}$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\begin{aligned}\int q^3 - 6q^2 dq &= \int q^3 dq - \int 6q^2 dq \\ &= \int q^3 dq - 6 \int q^2 dq \\ &= \frac{1}{4}q^4 - 6\frac{1}{3}q^3 + c\end{aligned}$$



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Integrate

(A) x^5

(B) t^8

(C) $12x^3$

(D) $q^3 - 6q^2$

$$\begin{aligned}\int q^3 - 6q^2 \, dq &= \int q^3 \, dq - \int 6q^2 \, dq \\ &= \int q^3 \, dq - 6 \int q^2 \, dq \\ &= \frac{1}{4}q^4 - 6\frac{1}{3}q^3 + c \\ &= \frac{1}{4}q^4 - 2q^3 + c.\end{aligned}$$



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Integrate

$$12e^{0.2t}.$$



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Integrate

$$12e^{0.2t}.$$

$$\int 12e^{0.2t} dt =$$



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Integrate

$$12e^{0.2t}.$$

$$\int 12e^{0.2t} dt = 12 \int e^{\frac{t}{5}} dt$$



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Integrate

$$12e^{0.2t}.$$

$$\begin{aligned}\int 12e^{0.2t} dt &= 12 \int e^{\frac{t}{5}} dt \\ &= 12 \left(5e^{\frac{t}{5}} \right) + c\end{aligned}$$



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Integrate

$$12e^{0.2t}.$$

$$\begin{aligned}\int 12e^{0.2t} dt &= 12 \int e^{\frac{t}{5}} dt \\ &= 12 \left(5e^{\frac{t}{5}} \right) + c \\ &= 60e^{0.2t} + c.\end{aligned}$$



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What is the indefinite integral of $\frac{1}{x^n}$?



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What is the indefinite integral of $\frac{1}{x^n}$? For $n < 1$, observe that



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What is the indefinite integral of $\frac{1}{x^n}$? For $n < 1$, observe that

$$\frac{d}{dx} \left[\frac{1}{-n+1} x^{-n+1} \right] =$$



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What is the indefinite integral of $\frac{1}{x^n}$? For $n < 1$, observe that

$$\frac{d}{dx} \left[\frac{1}{-n+1} x^{-n+1} \right] = \frac{1}{-n+1} \frac{d}{dx} x^{-n+1}$$



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What is the indefinite integral of $\frac{1}{x^n}$? For $n < 1$, observe that

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Hence

$$\int \frac{dx}{x^n} =$$



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What is the indefinite integral of $\frac{1}{x^n}$? For $n < 1$, observe that

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Hence

$$\int \frac{dx}{x^n} = \frac{1}{-n+1} x^{-n+1} + c$$



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What is the indefinite integral of $\frac{1}{x^n}$? For $n < 1$, observe that

$$\begin{aligned}\frac{d}{dx} \left[\frac{1}{-n+1} x^{-n+1} \right] &= \frac{1}{-n+1} \frac{d}{dx} x^{-n+1} \\ &= \frac{1}{-n+1} (-n+1) x^{-n} \\ &= x^{-n} \\ &= \frac{1}{x^n}\end{aligned}$$

Hence

$$\int \frac{dx}{x^n} = \frac{1}{-n+1} x^{-n+1} + c = \frac{1}{(-n+1)x^{n-1}} + c$$



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Integrate

$$\frac{1}{x^3}$$



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Integrate

$$\frac{1}{x^3}$$

$$\int \frac{dx}{x^3} =$$



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Integrate

$$\frac{1}{x^3}$$

$$\int \frac{dx}{x^3} = \int x^{-3} dx$$



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Integrate

$$\frac{1}{x^3}$$

$$\begin{aligned}\int \frac{dx}{x^3} &= \int x^{-3} dx \\ &= \frac{1}{-2} x^{-2} + c\end{aligned}$$



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Integrate

$$\frac{1}{x^3}$$

$$\begin{aligned}\int \frac{dx}{x^3} &= \int x^{-3} dx \\ &= \frac{1}{-2} x^{-2} + c \\ &= \frac{-1}{2x^2} + c.\end{aligned}$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$

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- When $n = 1$, the previous method fails because $1/(-n + 1)$ is undefined.



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- When $n = 1$, the previous method fails because $1/(-n + 1)$ is undefined.
- We observe that



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- When $n = 1$, the previous method fails because $1/(-n + 1)$ is undefined.
- We observe that

$$\frac{d}{dx} \ln(x) = \frac{1}{x},$$



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- When $n = 1$, the previous method fails because $1/(-n + 1)$ is undefined.
- We observe that

$$\frac{d}{dx} \ln(x) = \frac{1}{x},$$

so we would expect

$$\int \frac{dx}{x} = \ln(x) + c.$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$

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- When $n = 1$, the previous method fails because $1/(-n + 1)$ is undefined.
- We observe that

$$\frac{d}{dx} \ln(x) = \frac{1}{x},$$

so we would expect

$$\int \frac{dx}{x} = \ln(x) + c.$$

- This isn't quite true.



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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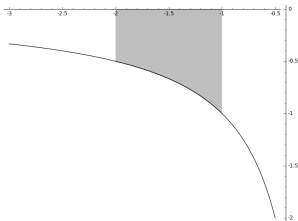
Let $F(x)$ be an antiderivative of $1/x$.



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

Let $F(x)$ be an antiderivative of $1/x$. Since this function is continuous away from $x = 0$, we could ask:

What is the area between $1/x$ and the x -axis from $x = -2$ to $x = -1$?



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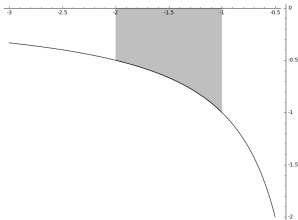
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Let $F(x)$ be an antiderivative of $1/x$. Since this function is continuous away from $x = 0$, we could ask:

What is the area between $1/x$ and the x -axis from $x = -2$ to $x = -1$?



By the Fundamental Theorem of Calculus, this is



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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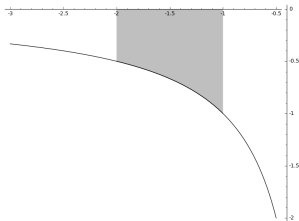
THE INDEFINITE INTEGRAL

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Let $F(x)$ be an antiderivative of $1/x$. Since this function is continuous away from $x = 0$, we could ask:

What is the area between $1/x$ and the x -axis from $x = -2$ to $x = -1$?



By the Fundamental Theorem of Calculus, this is

$$\int_{-2}^{-1} \frac{dx}{x} = F(-1) - F(-2).$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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Since $\ln(x)$ is only defined for **positive** values of x ,
 $F(x) \neq \ln(x)$. We can fix this by taking $F(x) = \ln|x|$.



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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Since $\ln(x)$ is only defined for **positive** values of x , $F(x) \neq \ln(x)$. We can fix this by taking $F(x) = \ln|x|$. Recall that

$$|x| = \begin{cases} x & \text{if } 0 \leq x, \\ -x & \text{if } x < 0 \end{cases} .$$



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$$|x| = \begin{cases} x & \text{if } 0 \leq x, \\ -x & \text{if } x < 0 \end{cases} .$$

For $x < 0$ we have

$$\frac{d}{dx} |x| =$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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For $x < 0$ we have

$$\frac{d}{dx} |x| = \frac{d}{dx} (-x)$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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For $x < 0$ we have

$$\frac{d}{dx} |x| = \frac{d}{dx} (-x) = -1.$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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NEGATIVE
EXPONENTS

Since $\ln(x)$ is only defined for **positive** values of x , $F(x) \neq \ln(x)$. We can fix this by taking $F(x) = \ln|x|$. Recall that

$$|x| = \begin{cases} x & \text{if } 0 \leq x, \\ -x & \text{if } x < 0 \end{cases}.$$

For $x < 0$ we have

$$\frac{d}{dx} |x| = \frac{d}{dx} (-x) = -1.$$

For $x > 0$ we have

$$\frac{d}{dx} |x| =$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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By the Chain Rule,



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$$\frac{d}{dx} \ln|x| =$$



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By the Chain Rule,

$$\frac{d}{dx} \ln|x| = \frac{\frac{d}{dx}|x|}{|x|}$$



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By the Chain Rule,

$$\begin{aligned}\frac{d}{dx} \ln|x| &= \frac{\frac{d}{dx} |x|}{|x|} \\ &= \begin{cases} \frac{1}{x} & \text{if } 0 < x \\ \frac{-1}{-x} = \frac{1}{x} & \text{if } x < 0. \end{cases}\end{aligned}$$



THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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THE INDEFINITE INTEGRAL OF $\frac{1}{x}$ (CONT.)

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Therefore

$$\int \frac{dx}{x} = \ln|x| + c.$$