Math 122

Farman

4.3: Global Maxima and Minima

2.5: Marginal Cost and Revenue

4.4: Profit, Cost, and Revenue

Maximizing Profit

Maximizing Revenue

Math 122

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Calculus for Business Administration and Social Sciences

Outline

1. 4.3: Global Maxima and Minima

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Theorem 1

If f is a continuous function defined on a closed interval, $[a, b]$, then f has a global minimum and a global maximum on $[a, b]$.

Example

Find the global extrema of $f(x) = x^3 - 9x^2 - 48x + 52$ on $[-5, 14]$.

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Maximum: (14,360).
Minimum: (8,-396).

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Remark 1

Critical points occur whenever marginal cost equal marginal revenue, or one of marginal cost/revenue is undefined.

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Find the quantity which maximizes profit for the given revenue and cost functions on $[0, 1000]$

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$$q = \frac{-(5 - 1.11)}{2(-0.003)} = 650$$

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At a price of \$80 for a half day trip, a white water rafting company attracts 300 customers. Every \$5 decrease in price attracts an additional 30 customers. What price should the company charge per trip to maximize revenue?

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$$\frac{-780}{2(-6)} = \frac{780}{12} = 65$$

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$$R(65) = \$25,350.$$