Math 122

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Calculus for Business Administration and Social Sciences

OUTLINE

1. 3.3: THE CHAIN RULE

Outline

The Chain Rule

Theorem 1

Let f and g be differentiable functions such that $f \circ g(x)$ is well-defined.

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Let f and g be differentiable functions such that $f \circ g(x)$ is well-defined. The derivative of the composition is given by

$$(f \circ g)'(x) = \left(f' \circ g(x)\right) \cdot g'(x).$$

Let $P(t) = P_0 a^t$.

The Derivative of Arbitrary Exponentials

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Let $f(t) = P_0 e^t$ and let $g(t) = \ln(a)t$ so

$(f \circ g)(t)$

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Hence

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$$\begin{aligned} P'(t) &= f' \circ g(t) \cdot g'(t) \\ &= P_0 e^{\ln(a)t} \cdot \ln(a) \\ &= P_0 a^t \cdot \ln(a) \\ &= \ln(a)P(t). \end{aligned}$$

Example

Differentiate $(x + 5)^2$.

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To differentiate g, we need to use the Chain Rule!

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To differentiate g, we need to use the Chain Rule!
If we let $h(t) = \underline{\qquad}$ and $k(t) = \underline{\qquad\quad}$, then $h \circ k(t) = \left(\ln\left(2t^2+3\right)\right)^2$.

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- $h'(t) = \frac{1}{t}$.

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- $h' \circ k(t) = h'(k(t)) = h'(2t^2+3) = \frac{1}{2t^2+3}$.
- Therefore by the Chain Rule,

$$g'(t) = \frac{1}{2t^2+3} \cdot 4t = \frac{4t}{2t^2+3}.$$

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Product Rule

If f and g are differentiable functions, then

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$$

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Differentiate

(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

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&= 2xe^{2x} + 2x^2 e^{2x} \\
&= 2xe^{2x}(1+x)
\end{aligned}$$

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Example

Differentiate

(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\begin{aligned}\frac{d}{dx}\left((3x^2+5x)e^x\right) &= \frac{d}{dx}(3x^2+5x)\cdot e^x + (3x^2+5x)\cdot\frac{d}{dx}e^x \\ &= (6x+5)e^x + (3x^2+5x)e^x\end{aligned}$$

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(A) $x^2 e^{2x}$ (B) $t^3 \ln(t+1)$ (C) $(3x^2+5x)e^x$

$$\begin{aligned} \frac{d}{dx}\left((3x^2+5x)e^x\right) &= \frac{d}{dx}(3x^2+5x)\cdot e^x + (3x^2+5x)\cdot\frac{d}{de^x} \\ &= (6x+5)e^x + (3x^2+5x)e^x \\ &= e^x(6x+5+3x^2+5x) \end{aligned}$$

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\frac{d}{dx}\left((3x^2+5x)e^x\right) &= \frac{d}{dx}(3x^2+5x)\cdot e^x + (3x^2+5x)\cdot\frac{d}{de^x} \\
&= (6x+5)e^x + (3x^2+5x)e^x \\
&= e^x(6x+5+3x^2+5x) \\
&= e^x(3x^2+11x+5).
\end{aligned}$$

EXAMPLE

Differentiate $\frac{e^{2t}}{t}$.

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\frac{\mathrm{d}}{\mathrm{d}t}\frac{e^{2t}}{t} &= \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{2t}\right)\cdot t - e^{2t}\cdot\frac{\mathrm{d}}{\mathrm{d}t}(t)}{t^2} \\
&= \frac{\frac{\mathrm{d}}{\mathrm{d}t}(2t)\cdot te^{2t} - e^{2t}(1)}{t^2} \\
&= \frac{2te^{2t} - e^{2t}}{t^2} \\
&= \frac{(2t-1)e^{2t}}{t^2}
\end{aligned}$$

Example

A product's price, p, is given by

$$p(q) = 80e^{-0.003q},$$

where q is the quantity sold.

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(B) How does revenue vary with respect to quantity?

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$$\begin{aligned} \frac{d}{dq}R(q) &= \frac{d}{dq}(80qe^{-0.003q}) \\ &= 80\frac{d}{dq}qe^{-0.003q} \end{aligned}$$

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\frac{\mathrm{d}}{\mathrm{d}q}R(q) &= \frac{\mathrm{d}}{\mathrm{d}q}(80qe^{-0.003q}) \\
&= 80\frac{\mathrm{d}}{\mathrm{d}q}qe^{-0.003q} \\
&= 80(e^{-0.003q} + q(-0.003)e^{-0.003q})
\end{aligned}$$

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&= 80\frac{\mathrm{d}}{\mathrm{d}q}qe^{-0.003q} \\
&= 80(e^{-0.003q} + q(-0.003)e^{-0.003q}) \\
&= 80e^{-0.003q}(1 - 0.003q).
\end{aligned}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

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$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right)$$

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Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) = \frac{10x(x^3+1) - 5x^2(3x^2)}{(x^3+1)^2}$$

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(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) = \frac{10x(x^3+1)-5x^2(3x^2)}{(x^3+1)^2} = \frac{10x^4+10x-15x^4}{(x^3+1)^2}$$

Example

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(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\begin{aligned}\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) &= \frac{10x(x^3+1)-5x^2(3x^2)}{(x^3+1)^2}\\ &= \frac{10x^4+10x-15x^4}{(x^3+1)^2}\\ &= \frac{-5x^4+10}{(x^3+1)^2}.\end{aligned}$$

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Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1+e^x}$$

Example

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$$\frac{d}{dx}\frac{1}{1+e^x} = \frac{0(1+e^x)-(1)e^x}{(1+e^x)^2}$$

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Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\frac{1}{1+e^x} = \frac{0(1+e^x)-(1)e^x}{(1+e^x)^2} = \frac{-e^x}{(1+e^x)^2}.$$

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(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

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$$\frac{d}{dx}\frac{e^x}{x^2} = \frac{e^x x^2 - e^x(2x)}{x^4}$$

Example

Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\frac{d}{dx}\frac{e^x}{x^2} = \frac{e^x x^2 - e^x(2x)}{x^4} = \frac{e^x(x^2-2x)}{x^4}$$

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Differentiate

(A) $\dfrac{5x^2}{x^3+1}$ (B) $\dfrac{1}{1+e^x}$ (C) $\dfrac{e^x}{x^2}$.

$$\begin{aligned}\frac{d}{dx}\frac{e^x}{x^2} &= \frac{e^x x^2 - e^x(2x)}{x^4} \\ &= \frac{e^x(x^2-2x)}{x^4} \\ &= \frac{e^x(x)(x-2)}{x^4}\end{aligned}$$

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Example

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- $f(2) = 1$,
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Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

Example

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- $f(2) = 1$,
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$$h'(2) =$$

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- $f(2) = 1$,
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Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$h'(2) = f'(2)g(2) + f(2)g'(2)$$

Example

Assume

- $f(2) = 1$,
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- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$\begin{aligned} h'(2) &= f'(2)g(2) + f(2)g'(2) \\ &= 5(3) + 1(6) \end{aligned}$$

Example

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- $f(2) = 1$,
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Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$\begin{aligned} h'(2) &= f'(2)g(2) + f(2)g'(2) \\ &= 5(3) + 1(6) \\ &= 21. \end{aligned}$$

Example

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(B) $k'(2)$.

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Assume

- $f(2) = 1$,
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- $g(2) = 3$,
- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$k'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2}$$

Example

Assume

- $f(2) = 1$,
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- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$,

(B) $k'(2)$.

$$k'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} = \frac{5(3) - 1(6)}{3^2}$$

EXAMPLE

Assume

- $f(2) = 1$,
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- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$, (B) $k'(2)$.

$$\begin{aligned} k'(2) &= \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} \\ &= \frac{5(3) - 1(6)}{3^2} \\ &= \frac{15 - 6}{9} \end{aligned}$$

Example

Assume

- $f(2) = 1$,
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- $g'(2) = 6$.

Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

(A) $h'(2)$, (B) $k'(2)$.

$$k'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} = \frac{5(3) - 1(6)}{3^2} = \frac{15 - 6}{9} = \frac{9}{9}$$

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Let $h(x) = f(x)g(x)$ and $k(x) = f(x)/g(x)$. Find

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(B) $k'(2)$.

$$\begin{aligned} k'(2) &= \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} \\ &= \frac{5(3) - 1(6)}{3^2} \\ &= \frac{15 - 6}{9} = \frac{9}{9} = 1. \end{aligned}$$