Math 122

Blake Farman [1]

[1] University of South Carolina, Columbia, SC USA

Calculus for Business Administration and Social Sciences

Outline

Definition

Definition 1

- The *doubling time* of an exponentially increasing quantity is the time required for the quantity to double.

Definition

Definition 1

- The *doubling time* of an exponentially increasing quantity is the time required for the quantity to double.
- The *half-life* of an exponentially decaying quantity is the time required for the quantity to be reduced by a factor of one half.

Doubling Time

Every exponentially increasing function, $P(t) = P_0a^t$, has a fixed doubling time, d.

Doubling Time

Every exponentially increasing function, $P(t) = P_0 a^t$, has a fixed doubling time, d. Take $d = \log_a(2)$.

Doubling Time

Every exponentially increasing function, $P(t) = P_0 a^t$, has a fixed doubling time, d. Take $d = \log_a(2)$. Then

$$P(t+d) = P_0 a^{t+d}$$

Doubling Time

Every exponentially increasing function, $P(t) = P_0 a^t$, has a fixed doubling time, d. Take $d = \log_a(2)$. Then

$$\begin{aligned} P(t+d) &= P_0 a^{t+d} \\ &= P_0 a^t a^d \end{aligned}$$

Doubling Time

Every exponentially increasing function, $P(t) = P_0 a^t$, has a fixed doubling time, d. Take $d = \log_a(2)$. Then

$$\begin{aligned} P(t+d) &= P_0 a^{t+d} \\ &= P_0 a^t a^d \\ &= P_0 a^t a^{\log_a(2)} \end{aligned}$$

Doubling Time

Every exponentially increasing function, $P(t) = P_0 a^t$, has a fixed doubling time, d. Take $d = \log_a(2)$. Then

$$\begin{aligned} P(t+d) &= P_0 a^{t+d} \\ &= P_0 a^t a^d \\ &= P_0 a^t a^{\log_a(2)} \\ &= 2P_0 a^t \end{aligned}$$

Doubling Time

Every exponentially increasing function, $P(t) = P_0 a^t$, has a fixed doubling time, d. Take $d = \log_a(2)$. Then

$$\begin{aligned} P(t+d) &= P_0 a^{t+d} \\ &= P_0 a^t a^d \\ &= P_0 a^t a^{\log_a(2)} \\ &= 2P_0 a^t \\ &= 2P(t). \end{aligned}$$

Half-Life

Similarly, every exponentially decreasing function, $P(t) = P_0 a^t$, has a fixed half-life, h.

Half-Life

Similarly, every exponentially decreasing function, $P(t) = P_0 a^t$, has a fixed half-life, h. Take

$$h = \log_a \left(\frac{1}{2}\right) = -\log_a(2).$$

Half-Life

Similarly, every exponentially decreasing function, $P(t) = P_0 a^t$, has a fixed half-life, h. Take

$$h = \log_a \left(\frac{1}{2}\right) = -\log_a(2).$$

Then

$$P(t+h) = P_0 a^{t+h}$$

Similarly, every exponentially decreasing function, $P(t) = P_0 a^t$, has a fixed half-life, h. Take

$$h = \log_a\left(\frac{1}{2}\right) = -\log_a(2).$$

Then

$$\begin{aligned} P(t+h) &= P_0 a^{t+h} \\ &= P_0 a^t a^h \end{aligned}$$

Half-Life

Similarly, every exponentially decreasing function, $P(t) = P_0 a^t$, has a fixed half-life, h. Take

$$h = \log_a\left(\frac{1}{2}\right) = -\log_a(2).$$

Then

$$\begin{aligned} P(t+h) &= P_0 a^{t+h} \\ &= P_0 a^t a^h \\ &= P_0 a^t a^{-\log_a(2)} \end{aligned}$$

Half-Life

Similarly, every exponentially decreasing function, $P(t) = P_0 a^t$, has a fixed half-life, h. Take

$$h = \log_a\left(\frac{1}{2}\right) = -\log_a(2).$$

Then

$$\begin{aligned}
P(t+h) &= P_0 a^{t+h} \\
&= P_0 a^t a^h \\
&= P_0 a^t a^{-\log_a(2)} \\
&= \frac{1}{2} P_0 a^t
\end{aligned}$$

Half-Life

Similarly, every exponentially decreasing function, $P(t) = P_0 a^t$, has a fixed half-life, h. Take

$$h = \log_a\left(\frac{1}{2}\right) = -\log_a(2).$$

Then

$$\begin{aligned} P(t+h) &= P_0 a^{t+h} \\ &= P_0 a^t a^h \\ &= P_0 a^t a^{-\log_a(2)} \\ &= \frac{1}{2} P_0 a^t \\ &= \frac{1}{2} P(t). \end{aligned}$$

Computing Doubling Time/Half-Life

To approximate the value of the doubling time with a calculator:

$$d = log_a(2) = \frac{\ln(2)}{\ln(a)}$$

and

$$h = -\log_a(2) = -\frac{\ln(2)}{\ln(a)}.$$

Example

Raditiation from an iodine source decays at a continuous hourly rate of $k = -0.004$.

Example

Raditiation from an iodine source decays at a continuous hourly rate of $k = -0.004$. If the radiation level at a spill is about 2.4 millirems/hour:

Example

Raditiation from an iodine source decays at a continuous hourly rate of $k = -0.004$. If the radiation level at a spill is about 2.4 millirems/hour:

(A) What was the radiation level 24 hours later?

EXAMPLE

Raditiation from an iodine source decays at a continuous hourly rate of $k = -0.004$. If the radiation level at a spill is about 2.4 millirems/hour:

(A) What was the radiation level 24 hours later?

(B) How long will it take for the radiation levels to decay to the maximum acceptable radiation level of 0.6 millirems/hour set by the EPA?

Example (Cont.)

(a) The radiation level 24 hours later is

$$R(24) = 2.4e^{-0.004 \cdot 24} \approx 2.18 \text{ millirems/hour.}$$

Example (cont.)

(a) The radiation level 24 hours later is

$$R(24) = 2.4e^{-0.004 \cdot 24} \approx 2.18 \text{ millirems/hour.}$$

(b) Solve the equation below for t:

Example (cont.)

(a) The radiation level 24 hours later is

$$R(24) = 2.4e^{-0.004 \cdot 24} \approx 2.18 \text{ millirems/hour.}$$

(b) Solve the equation below for t:

$$0.6 = 2.4e^{-0.004t}$$

Example (Cont.)

(a) The radiation level 24 hours later is

$$R(24) = 2.4e^{-0.004 \cdot 24} \approx 2.18 \text{ millirems/hour}.$$

(b) Solve the equation below for t:

$$\begin{aligned} 0.6 &= 2.4e^{-0.004t} \\ \Rightarrow e^{-0.004t} &= \frac{2.4}{0.6} = \frac{1}{4} \end{aligned}$$

Example (Cont.)

(a) The radiation level 24 hours later is

$$R(24) = 2.4e^{-0.004\cdot 24} \approx 2.18 \text{ millirems/hour.}$$

(b) Solve the equation below for t:

$$\begin{aligned} 0.6 &= 2.4e^{-0.004t} \\ \Rightarrow e^{-0.004t} &= \frac{2.4}{0.6} = \frac{1}{4} \\ \Rightarrow -0.004t &= \ln\left(\frac{1}{4}\right) = -\ln(4) \end{aligned}$$

Example (Cont.)

(a) The radiation level 24 hours later is

$$R(24) = 2.4e^{-0.004 \cdot 24} \approx 2.18 \text{ millirems/hour.}$$

(b) Solve the equation below for t:

$$\begin{aligned} 0.6 &= 2.4e^{-0.004t} \\ \Rightarrow e^{-0.004t} &= \frac{2.4}{0.6} = \frac{1}{4} \\ \Rightarrow -0.004t &= \ln\left(\frac{1}{4}\right) = -\ln(4) \\ \Rightarrow t &= \frac{1}{0.004}\ln(4) \approx 346.57 \text{ hours.} \end{aligned}$$

Example (Cont.)

(A) The radiation level 24 hours later is

$$R(24) = 2.4e^{-0.004\cdot 24} \approx 2.18 \text{ millirems/hour.}$$

(B) Solve the equation below for t:

$$\begin{aligned} 0.6 &= 2.4e^{-0.004t} \\ \Rightarrow e^{-0.004t} &= \frac{2.4}{0.6} = \frac{1}{4} \\ \Rightarrow -0.004t &= \ln\left(\frac{1}{4}\right) = -\ln(4) \\ \Rightarrow t &= \frac{1}{0.004}\ln(4) \approx 346.57 \text{ hours.} \end{aligned}$$

Therefore, it will take approximately $346.57/24 = 14.4$ days.

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009.

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$.

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$. If we assume that $P(t) = 19.5e^{kt}$, then

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$. If we assume that $P(t) = 19.5e^{kt}$, then

$$39 = 19.5e^{25k}$$

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$. If we assume that $P(t) = 19.5e^{kt}$, then

$$\begin{aligned} 39 &= 19.5e^{25k} \\ \Rightarrow \frac{39}{19.5} &= 2 = e^{25k} \end{aligned}$$

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$. If we assume that $P(t) = 19.5e^{kt}$, then

$$\begin{aligned} 39 &= 19.5e^{25k} \\ \Rightarrow \frac{39}{19.5} &= 2 = e^{25k} \\ \Rightarrow \ln(2) &= \ln(e^{25k}) = 25k \end{aligned}$$

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$. If we assume that $P(t) = 19.5e^{kt}$, then

$$\begin{aligned} 39 &= 19.5e^{25k} \\ \Rightarrow \frac{39}{19.5} &= 2 = e^{25k} \\ \Rightarrow \ln(2) &= \ln(e^{25k}) = 25k \\ \Rightarrow k &= \frac{ln(2)}{25} \end{aligned}$$

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$. If we assume that $P(t) = 19.5e^{kt}$, then

$$\begin{aligned} 39 &= 19.5e^{25k} \\ \Rightarrow \frac{39}{19.5} &= 2 = e^{25k} \\ \Rightarrow \ln(2) &= \ln(e^{25k}) = 25k \\ \Rightarrow k &= \frac{ln(2)}{25} \approx 0.028. \end{aligned}$$

The population of Kenya was about 19.5 million in 1984 and 39 million in 2009. Find, assuming exponential growth, a function of t years since 1984 modeling the population.

We are given $P_0 = 19.5$ and $P(25) = 39$. If we assume that $P(t) = 19.5e^{kt}$, then

$$\begin{aligned} 39 &= 19.5e^{25k} \\ \Rightarrow \frac{39}{19.5} &= 2 = e^{25k} \\ \Rightarrow \ln(2) &= \ln(e^{25k}) = 25k \\ \Rightarrow k &= \frac{ln(2)}{25} \approx 0.028. \end{aligned}$$

Therefore

$$P(t) \approx 19.5e^{0.28t}.$$

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere.

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere. The quantity of ozone, $Q(t)$, decays exponentially at a continuous rate of 0.25% per year.

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere. The quantity of ozone, $Q(t)$, decays exponentially at a continuous rate of 0.25% per year. What is the half-life of ozone?

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere. The quantity of ozone, $Q(t)$, decays exponentially at a continuous rate of 0.25% per year. What is the half-life of ozone?

The half life is given by

$$\log_{e^k}(2) = -\frac{\ln(2)}{\ln(e^k)}$$

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere. The quantity of ozone, $Q(t)$, decays exponentially at a continuous rate of 0.25% per year. What is the half-life of ozone?
The half life is given by

$$\begin{aligned} \log_{e^k}(2) &= -\frac{\ln(2)}{\ln(e^k)} \\ &= -\frac{ln(2)}{k} \end{aligned}$$

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere. The quantity of ozone, $Q(t)$, decays exponentially at a continuous rate of 0.25% per year. What is the half-life of ozone?

The half life is given by

$$\begin{aligned} \log_{e^k}(2) &= -\frac{\ln(2)}{\ln(e^k)} \\ &= -\frac{ln(2)}{k} \\ &= -\frac{ln(2)}{-\frac{1}{400}} \end{aligned}$$

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere. The quantity of ozone, $Q(t)$, decays exponentially at a continuous rate of 0.25% per year. What is the half-life of ozone?

The half life is given by

$$\begin{aligned}
\log_{e^k}(2) &= -\frac{\ln(2)}{\ln(e^k)} \\
&= -\frac{ln(2)}{k} \\
&= -\frac{ln(2)}{-\frac{1}{400}} \\
&= 400\ln(2)
\end{aligned}$$

The release of chlorofluorocarbons (CFCs) used in air conditioners and household aerosols destroys the ozone layer in the upper atmosphere. The quantity of ozone, $Q(t)$, decays exponentially at a continuous rate of 0.25% per year. What is the half-life of ozone?

The half life is given by

$$\begin{aligned} \log_{e^k}(2) &= -\frac{\ln(2)}{\ln(e^k)} \\ &= -\frac{ln(2)}{k} \\ &= -\frac{ln(2)}{-\frac{1}{400}} \\ &= 400\ln(2) \approx 277 \text{ years.} \end{aligned}$$

Compound Interest

Assume a sum of money P_0 is deposited in an account paying interest at a rate of r yearly, compounded n times per year.

Compound Interest

Assume a sum of money P_0 is deposited in an account paying interest at a rate of r yearly, compounded n times per year. This means that each compounding period, the account earns interest on the balance at a rate of r/n.

Compound Interest

Assume a sum of money P_0 is deposited in an account paying interest at a rate of r yearly, compounded n times per year. This means that each compounding period, the account earns interest on the balance at a rate of r/n. What is the balance of the account after t years?

Compounding Interest (Cont.)

Consider the table:

Compounding Period	Account Balance

Compounding Interest (Cont.)

Consider the table:

Compounding Period	Account Balance
1	$P_0\left(1+\frac{r}{n}\right)$

Compounding Interest (cont.)

Consider the table:

Compounding Period	Account Balance
1	$P_0\left(1+\frac{r}{n}\right)$
2	$P_0\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^2$

Compounding Interest (cont.)

Consider the table:

Compounding Period	Account Balance
1	$P_0\left(1+\frac{r}{n}\right)$
2	$P_0\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^2$
3	$P_0\left(1+\frac{r}{n}\right)^2\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^3$

Compounding Interest (cont.)

Consider the table:

Compounding Period	Account Balance
1	$P_0\left(1+\frac{r}{n}\right)$
2	$P_0\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^2$
3	$P_0\left(1+\frac{r}{n}\right)^2\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^3$
$\vdots$	$\vdots$

Compounding Interest (cont.)

Consider the table:

Compounding Period	Account Balance
1	$P_0\left(1+\frac{r}{n}\right)$
2	$P_0\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^2$
3	$P_0\left(1+\frac{r}{n}\right)^2\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^3$
$\vdots$	$\vdots$
n	$P_0\left(1+\frac{r}{n}\right)^n$

Compounding Interest (cont.)

Consider the table:

Compounding Period	Account Balance
1	$P_0\left(1+\frac{r}{n}\right)$
2	$P_0\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^2$
3	$P_0\left(1+\frac{r}{n}\right)^2\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^3$
$\vdots$	$\vdots$
n	$P_0\left(1+\frac{r}{n}\right)^n$

So at the end of the year, the balance will be $P_0\left(1+\frac{r}{n}\right)^n$.

Consider the table:

Compounding Period	Account Balance
1	$P_0\left(1+\frac{r}{n}\right)$
2	$P_0\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^2$
3	$P_0\left(1+\frac{r}{n}\right)^2\left(1+\frac{r}{n}\right)=P_0\left(1+\frac{r}{n}\right)^3$
$\vdots$	$\vdots$
n	$P_0\left(1+\frac{r}{n}\right)^n$

So at the end of the year, the balance will be $P_0\left(1+\frac{r}{n}\right)^n$. Continuing this way, the account balance after t years will be

$$P_0\left(1+\frac{r}{n}\right)^{nt}.$$

Doubling Time

Say you invest P_0 dollars at a rate of r per year, compounded n times.

Say you invest P_0 dollars at a rate of r per year, compounded n times. What is the doubling time?

Doubling Time

Say you invest P_0 dollars at a rate of r per year, compounded n times. What is the doubling time? The function for the account balance is

$$P_0\left(1+\frac{r}{n}\right)^{nt} = P_0\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t}.$$

Doubling Time

Say you invest P_0 dollars at a rate of r per year, compounded n times. What is the doubling time? The function for the account balance is

$$P_0\left(1+\frac{r}{n}\right)^{nt} = P_0\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t}.$$

Therefore the doubling time is

$$d = \log_{\left(1+\frac{r}{n}\right)^n}(2)$$

Doubling Time

Say you invest P_0 dollars at a rate of r per year, compounded n times. What is the doubling time? The function for the account balance is

$$P_0\left(1+\frac{r}{n}\right)^{nt}=P_0\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t}.$$

Therefore the doubling time is

$$d=\log_{\left(1+\frac{r}{n}\right)^n}(2)=\frac{\ln(2)}{\ln\left(\left(1+\frac{r}{n}\right)^n\right)}$$

Doubling Time

Say you invest P_0 dollars at a rate of r per year, compounded n times. What is the doubling time? The function for the account balance is

$$P_0\left(1+\frac{r}{n}\right)^{nt}=P_0\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t}.$$

Therefore the doubling time is

$$d=\log_{\left(1+\frac{r}{n}\right)^n}(2)=\frac{\ln(2)}{\ln\left(\left(1+\frac{r}{n}\right)^{n}\right)}=\frac{ln(2)}{n\ln\left(1+\frac{r}{n}\right)}.$$

Example

Say the interest rate is 2% and interest is compounded yearly.

Example

Say the interest rate is 2% and interest is compounded yearly. The expected doubling time is

$$d = \frac{\ln(2)}{\ln(1.02)}$$

Example

Say the interest rate is 2% and interest is compounded yearly. The expected doubling time is

$$d = \frac{\ln(2)}{\ln(1.02)} \approx 35 \text{ years.}$$

Example

Say the interest rate is 2% and interest is compounded yearly. The expected doubling time is

$$d = \frac{\ln(2)}{\ln(1.02)} \approx 35 \text{ years.}$$

Remark 1 ("Rule of 70")

When $r\%$ is very small,

$$\ln\left(1 + \frac{r}{100}\right) \approx \frac{r}{100}$$

and $\ln(2) \approx .7$, so the doubling rate is approximately

Example

Say the interest rate is 2% and interest is compounded yearly. The expected doubling time is

$$d = \frac{\ln(2)}{\ln(1.02)} \approx 35 \text{ years}.$$

Remark 1 ("Rule of 70")

When $r\%$ is very small,

$$\ln\left(1 + \frac{r}{100}\right) \approx \frac{r}{100}$$

and $\ln(2) \approx .7$, so the doubling rate is approximately

$$d = \frac{\ln(2)}{\ln\left(1 + \frac{r}{100}\right)}$$

Example

Say the interest rate is 2% and interest is compounded yearly. The expected doubling time is

$$d = \frac{\ln(2)}{\ln(1.02)} \approx 35 \text{ years.}$$

Remark 1 ("Rule of 70")

When $r\%$ is very small,

$$\ln\left(1 + \frac{r}{100}\right) \approx \frac{r}{100}$$

and $\ln(2) \approx .7$, so the doubling rate is approximately

$$d = \frac{\ln(2)}{\ln\left(1 + \frac{r}{100}\right)} \approx \frac{.7}{r/100}$$

Example

Say the interest rate is 2% and interest is compounded yearly. The expected doubling time is

$$d = \frac{\ln(2)}{\ln(1.02)} \approx 35 \text{ years.}$$

Remark 1 ("Rule of 70")

When $r\%$ is very small,

$$\ln\left(1 + \frac{r}{100}\right) \approx \frac{r}{100}$$

and $\ln(2) \approx .7$, so the doubling rate is approximately

$$d = \frac{\ln(2)}{\ln\left(1 + \frac{r}{100}\right)} \approx \frac{.7}{r/100} = \frac{70}{r}.$$

The method above is discrete.

Continuously Compounding Interest

The method above is discrete. If instead, we wish to compound interest at every instant, we get *continuously compounding interest*,

$$P(t) = P_0 e^{rt}.$$

Example

If $\$10,000$ is invested at 5% per year, compounded continuously, how long will it take to reach $\$15,000$?

Example

If $10,000 is invested at 5% per year, compounded continuously, how long will it take to reach $15,000? We want to solve the equation below for t:

$$P(t) \quad = \quad 10000e^{t/20} = 15000$$

Example

If $\$10,000$ is invested at 5% per year, compounded continuously, how long will it take to reach $\$15,000$? We want to solve the equation below for t:

$$\begin{aligned} P(t) &= 10000e^{t/20} = 15000 \\ \Rightarrow e^{t/20} &= \frac{15000}{10000} = \frac{3}{2} \end{aligned}$$

Example

If $\$10,000$ is invested at 5% per year, compounded continuously, how long will it take to reach $\$15,000$? We want to solve the equation below for t:

$$\begin{aligned} P(t) &= 10000e^{t/20} = 15000 \\ \Rightarrow e^{t/20} &= \frac{15000}{10000} = \frac{3}{2} \\ \Rightarrow t/20 &= \ln(e^{t/20}) = \ln\left(\frac{3}{2}\right) \end{aligned}$$

Example

If $\$10,000$ is invested at 5% per year, compounded continuously, how long will it take to reach $\$15,000$? We want to solve the equation below for t:

$$
\begin{aligned}
P(t) &= 10000e^{t/20} = 15000 \\
\Rightarrow e^{t/20} &= \frac{15000}{10000} = \frac{3}{2} \\
\Rightarrow t/20 &= \ln(e^{t/20}) = \ln\left(\frac{3}{2}\right) \\
\Rightarrow t &= 20\ln\left(\frac{3}{2}\right)
\end{aligned}
$$

Example

If $10,000 is invested at 5% per year, compounded continuously, how long will it take to reach $15,000? We want to solve the equation below for t:

$$\begin{aligned} P(t) &= 10000e^{t/20} = 15000 \\ \Rightarrow e^{t/20} &= \frac{15000}{10000} = \frac{3}{2} \\ \Rightarrow t/20 &= \ln(e^{t/20}) = \ln\left(\frac{3}{2}\right) \\ \Rightarrow t &= 20\ln\left(\frac{3}{2}\right) \\ &\approx 8 \text{ years.} \end{aligned}$$

Doubling Time

Say you invest P_0 dollars at a rate of $r\%$ per year compounding continuously.

Doubling Time

Say you invest P_0 dollars at a rate of $r\%$ per year compounding continuously. The account balance is given by the function

$$P_0 e^{\frac{r}{100}t} = P_0 (e^{\frac{r}{100}})^t.$$

Doubling Time

Say you invest P_0 dollars at a rate of $r\%$ per year compounding continuously. The account balance is given by the function

$$P_0 e^{\frac{r}{100}t} = P_0(e^{\frac{r}{100}})^t.$$

Hence the doubling time is given by

$$\log_{e^{\frac{r}{100}}}(2) = \frac{\ln(2)}{\ln(e^{\frac{r}{100}})}$$

Doubling Time

Say you invest P_0 dollars at a rate of $r\%$ per year compounding continuously. The account balance is given by the function

$$P_0 e^{\frac{r}{100}t} = P_0(e^{\frac{r}{100}})^t.$$

Hence the doubling time is given by

$$\log_{e^{\frac{r}{100}}}(2) = \frac{\ln(2)}{\ln(e^{\frac{r}{100}})} = \frac{ln(2)}{\frac{r}{100}}$$

Doubling Time

Say you invest P_0 dollars at a rate of $r\%$ per year compounding continuously. The account balance is given by the function

$$P_0 e^{\frac{r}{100}t} = P_0(e^{\frac{r}{100}})^t.$$

Hence the doubling time is given by

$$\log_{e^{\frac{r}{100}}}(2) = \frac{\ln(2)}{\ln(e^{\frac{r}{100}})} = \frac{ln(2)}{\frac{r}{100}} \approx \frac{70}{r}.$$