Math 122

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Calculus for Business Administration and Social Sciences

OUTLINE

1. 1.8: NEW FUNCTIONS FROM OLD
    - Function Composition
    - Scaling
    - Rigid Transformations

Function Composition

Definition 1

Given a function f and a function g such that the range of f is contained in the domain of g we can define the composition

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- $f(x) = x + 1$, and

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$$f \circ g(x) = \frac{1}{x - 1}.$$

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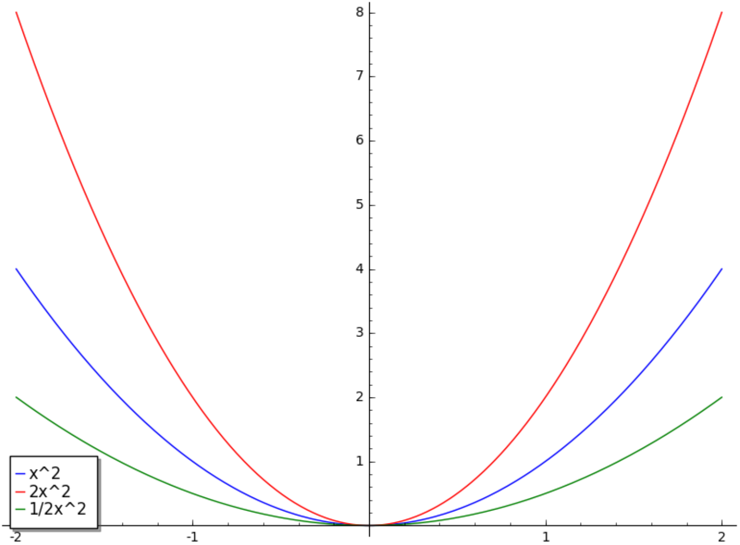
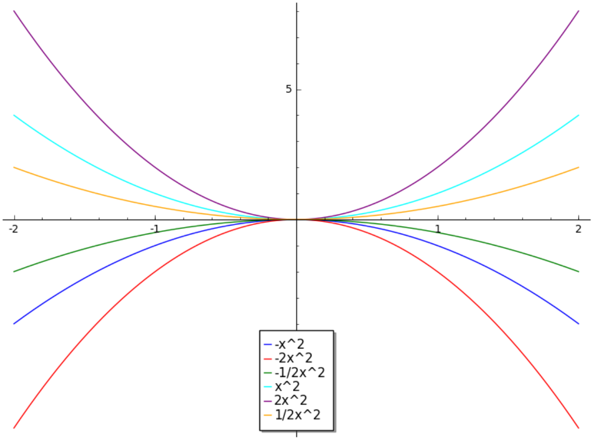
- a *vertical stretching* of the graph of $f(x)$ if $1 < a$
- a *vertical shrinking* of the graph of $f(x)$ if $a < 1$.

Examples



Reflection

The graph of $-f(x)$ is a reflection of $f(x)$ across the x-axis.



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- The graph of $f(x) - a$ is the graph of $f(x)$ shifted down a units.

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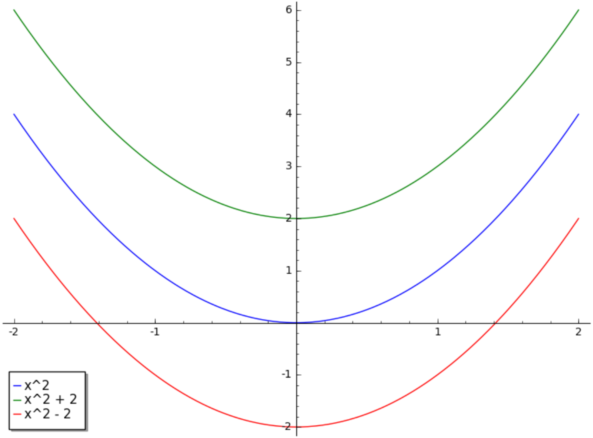

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- The graph of $f(x - a)$ is a horizontal shift of $f(x)$ by a units to the right.
- The graph of $f(x + a)$ is a horizontal shift of $f(x)$ by a units to the left.

Examples