# Math 122

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Calculus for Business Administration and Social Sciences

# Outline





1. 1.3: Average Rate of Change and Relative Change

# Outline

# Average Rate of Change

**Definition 1**

The *average rate of change* of a function $f$ on an interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{f(a) - f(b)}{a - b}.$$

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**Remark 1**

This is just the difference quotient from the last section.

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# EXAMPLE



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$$\frac{104 - 0}{2 - 0} = \frac{104}{2} = 52 \text{ mph.}$$

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From Columbia, it's about 104 miles to Charleston. If you make the drive in two hours, what was your average speed? Take Columbia to be distance zero, and mark the starting time at $t = 0$. The average speed is:

$$\frac{104 - 0}{2 - 0} = \frac{104}{2} = 52 \text{ mph.}$$

**Remark 2**

Note that this does not necessarily imply you drove 52 mph the entire time, but rather you averaged 52 mph.

# EXAMPLE

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# Relative Change



Given a quantity, $P$, the *relative change* of the quantity from $P$ to $P'$ is

$$\frac{P' - P}{P}.$$

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If gas costs \$2.25 and the price increases by \$2, then find the relative change in price.

$$\frac{4.25 - 2.25}{2.25} = \frac{2}{2.25} = \frac{2}{\frac{9}{4}} = \frac{8}{9} = 0.\overline{8}.$$

# Example

A pair of jeans costs 75.99 normally.

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A pair of jeans costs 75.99 normally. Today they are on sale for 52.99. What is the relative change in the price?

$$\frac{52.99 - 75.99}{75.99} = \frac{-23}{75.99} \approx -0.303.$$

Hence the price has been reduced by about 30%.

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The number of sales per week for the jeans above is normally 25.

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$$\frac{45-25}{25} = \frac{20}{25} = \frac{4}{5}.$$

# Example



The number of sales per week for the jeans above is normally 25. During the week the jeans are on sale, the number of weekly sales increases to 45. Find the relative change in weekly sales.

$$\frac{45-25}{25} = \frac{20}{25} = \frac{4}{5}.$$

Hence weekly sales have increased by 80%.

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- the profit from selling $q$ goods by $\pi(q)$.

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- The $\$24,000$ expenditue is called a *fixed cost*.
- The $7/radio in labour and material is called a *variable cost*.

# Linear Marginal Cost

**Definition 2**

For a linear cost function, the marginal cost is the cost to product one additional unit:

$$\frac{C(q+1)-C(q)}{(q+1)-q}=C(q+1)-C(q).$$

# Linear Marginal Cost

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$$\frac{C(q+1)-C(q)}{(q+1)-q} = C(q+1)-C(q).$$

**Remark 3**

This is just the slope of the linear cost function.

# Profit

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- Given a revenue and a cost function, the profit function is

$$\pi(q) = R(q) - C(q).$$

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- Given a revenue and a cost function, the profit function is
$$\pi(q) = R(q) - C(q).$$
- The *break-even* point is the quantity, $q$, for which
$$\pi(q) = 0$$
holds.

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$$\pi(q) = R(q) - C(q) = 15q - (7q + 24000)$$

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hold. Therefore the break-even point is

$$q = \frac{24000}{8} = 3000.$$

# Marginal Revenue

**Definition 4**

The *marginal revenue* for a linear revenue function is the revenue from selling one additional item,

$$\frac{R(q+1)-R(q)}{(q+1)-q} = R(q+1)-R(q).$$

# Marginal Revenue

**Definition 4**

The *marginal revenue* for a linear revenue function is the revenue from selling one additional item,

$$\frac{R(q+1)-R(q)}{(q+1)-q} = R(q+1)-R(q).$$

**Remark 4**

This is just the slope of the revenue function.

# Marginal Profit

**Definition 5**

The *marginal profit* for linear cost and revenue functions is the profit from selling one additional item

$$\frac{\pi(q+1)-\pi(q)}{(q+1)-q} = \pi(q+1)-\pi(q).$$

# Marginal Profit

## Definition 5

The *marginal profit* for linear cost and revenue functions is the profit from selling one additional item

$$\frac{\pi(q+1)-\pi(q)}{(q+1)-q} = \pi(q+1)-\pi(q).$$

## Remark 5

This is the slope of the revenue function less the slope of the cost function.