

## 11 Simplifying Algebraic Expressions

### Difference Quotients

$\frac{f(x+h)-f(x)}{h}$  - talk about secant lines, etc

E.g.: ① If  $f(x) = x^2$ , compute the difference quotient.

$$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{h(2x+h)}{h} = 2x+h$$

②  $f(x) = 2x^2 - 3x$

$$\begin{aligned} \frac{1}{h} (2(x+h)^2 - 3(x+h) - 2x^2 + 3x) &= \frac{1}{h} (2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x) \\ &= \frac{1}{h} (4xh + 2h^2 - 3h) \\ &= 4x + 2h - 3. \end{aligned}$$

③  $f(x) = \frac{1}{x}$ , compute the difference quotient

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = \frac{-1}{x(x+h)}$$

④  $f(x) = x^2 + \frac{1}{x}$

$$\begin{aligned} \frac{1}{h} ((x+h)^2 + \frac{1}{x+h} - x^2 - \frac{1}{x}) &= \frac{1}{h} (x^2 + 2xh + h^2 + \frac{1}{x+h} - x^2 - \frac{1}{x}) \\ &= \frac{1}{h} (2xh + h^2 + \frac{1}{x+h} - \frac{1}{x}) \\ &= \frac{1}{h} (2xh + h^2 + \frac{x-(x+h)}{x(x+h)}) \\ &= \frac{1}{h} (2xh + h^2 - \frac{h}{x(x+h)}) \\ &= 2x + h - \frac{1}{x(x+h)}. \end{aligned}$$