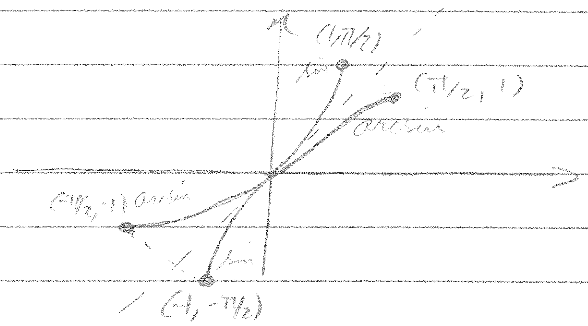
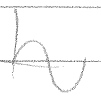
9 Inverse Trigonometric Functions

9.1 arcsin(x)

Note sin(x) fails the horizonal line test.
Restrict to $[-\pi/2, \pi/2]$.





Rmk a) $\sin^{-1}(x) \neq 1/\sin(x) = \csc(x)$.

b) domain of arcsin(x) is $[-1,1]$ and its range is $[-\pi/2, \pi/2]$.

c) Generally, when $x \in [-1,1]$, arcsin(x) is the number $y \in [-\pi/2, \pi/2]$ such that $\sin(y) = x$.

Eg: Find

a) $\arcsin(0) = 0$,

b) $\arcsin(1) = \pi/2$,

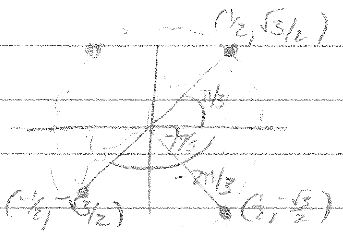
c) $\arcsin(1/2) = \pi/6$,

d) $\arcsin(-\sqrt{3}/2) = -\pi/3$,

e) arcsin(2) is undefined.

(d) $\sin(-2\pi/3) = -\sqrt{3}/2$

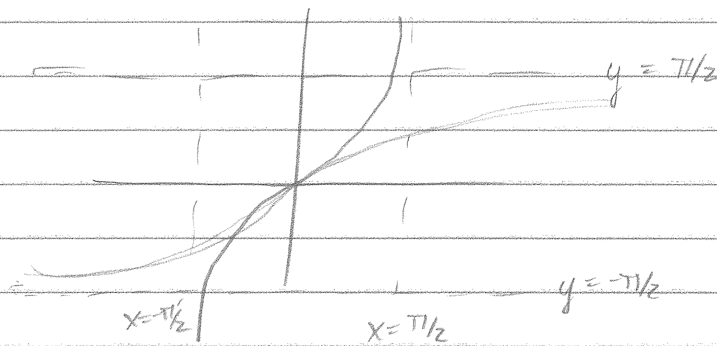
$-\frac{2\pi}{3} + \frac{\pi}{3} = -\pi/3$



? computed this

9.2 arctan(x) and arcsec(x)

Restrict $\tan(x)$ to $[-\pi/2, -\pi/2]$



Rmk $\arctan(x)$ is the $y \in (-\pi/2, \pi/2)$ such that $\tan(y) = x$.

Eg: a) $\arctan(0) = 0$ $(\sin(0) = 0)$

b) $\arctan(1) = \pi/4$ $(\sin(\pi/4) = \cos(\pi/4))$.

c) $\arctan(-\sqrt{3})$

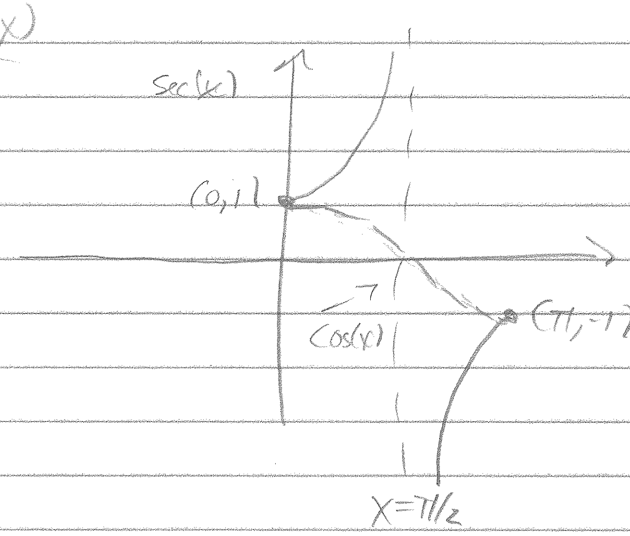
By (d) above

$$\tan(-\pi/3) = \frac{\sin(-\pi/3)}{\cos(-\pi/3)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}.$$

so

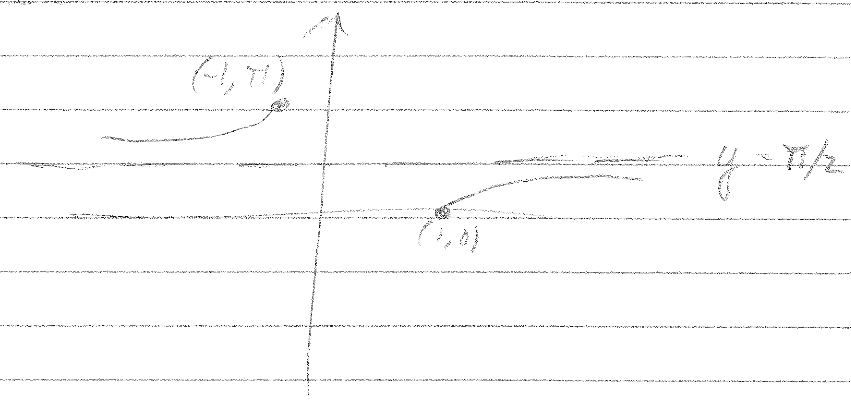
$$\arctan(-\sqrt{3}) = -\pi/3.$$

arcsec(x)



(Explain the asymptote at $x = \pi/2$)

arcsec(x)

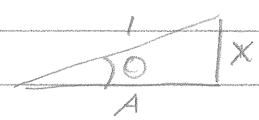


arcsec(x) is the number $y \in [0, \pi/2) \cup (\pi/2, \pi]$ with $\sec(y) = x$.

§3 Trig Identities

Eg. ① Simplify $\tan \circ \arcsin(x)$.

Let $\theta = \arcsin(x)$, so $\sin(\theta) = x$ provided $x \in [-1, 1]$. Make a triangle:



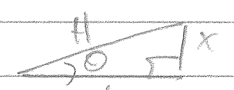
$A^2 + x^2 = 1 \Rightarrow A^2 = 1 - x^2$

$\Rightarrow A = \sqrt{1 - x^2}$ ($A > 0$ necessarily)

$\Rightarrow \tan(\theta) = \tan \circ \arcsin(x) = x / \sqrt{1 - x^2}$.

② Simplify $\cos \circ \arctan(x)$.

$\theta = \arctan(x) \Rightarrow x = \tan(\theta)$



$H^2 = x^2 + 1$

$\Rightarrow H = \sqrt{x^2 + 1}$

$\cos(\theta) = \frac{1}{\sqrt{x^2 + 1}}$

$\cos(\theta) = \cos \circ \arctan(x) = \frac{1}{\sqrt{x^2 + 1}}$