6. Exponential Functions

Def'n: An exponential function has the form $f(x) = a^x$ where $a > 0$ is a real number

Eg.: $f(x) = 2^x$

$f(1) = 2$, $f(2) = 4$, $f(3) = 8$, ...

$f(1/2) = \sqrt{2}$

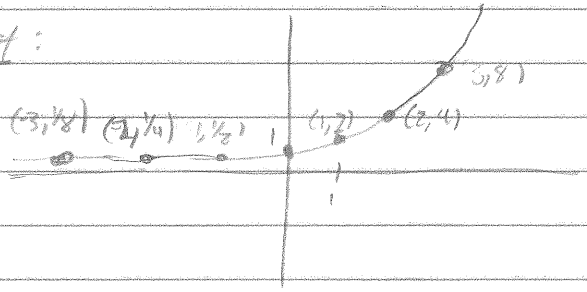
$f(-1) = 1/2$, $f(-2) = 1/4$, $f(-3) = 1/8$, ...

$f(16/5) = 2^{16/5} = 2^{3+1/5} = 2^3 2^{1/5} = 8\sqrt[5]{2}$.

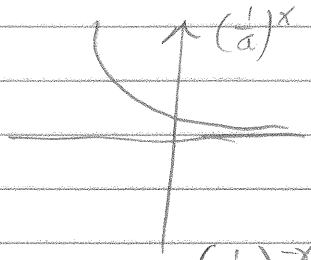
$3.2 = 3 + \frac{2}{10} = 3 + \frac{1}{5}$

$= \frac{15+1}{5} = \frac{16}{5}$

Plot:



Graph of $f(x) = a^x$, $a > 1$ as above. Steeper as $a \to \infty$.
Graph of $f(x) = (\frac{1}{a})^x$ the mirror image (across the y-axis.)



This is because $(\frac{1}{a})^{-x} = a^x$.

Rmk: If a is negative, this is no longer a real valued function: $a = -1$, $(-1)^{1/2} = i \notin \mathbb{R}$.