4.2 Lines and their Equations

Def^n. The slope of a line passing through the points (x_0, y_0) and (x_1, y_1)

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

Rmk This is independent of the choice of points on the line.

Point-Slope form

Given a point (x_0, y_0) and a slope the line passing through (x_0, y_0) with slope m is

$$y - y_0 = m(x - x_0).$$

Ex ① Find the line with slope -1 through $(-2, 1)$.

$y - 1 = (-1)(x - (-2))$

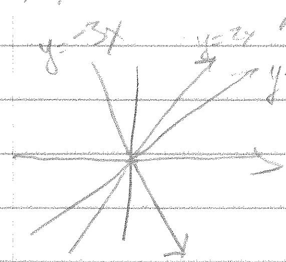
$\Rightarrow y - 1 = (-1)(x + 2)$. ▨

② Find the equation of the line through $(2, -5)$ with slope -3.

$y - (-5) = (-3)(x - 2)$

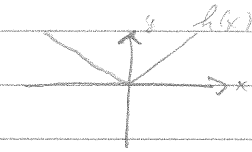
$\Rightarrow y + 5 = -3(x - 2)$ ▨

③ Find the lines through the origin with slopes 1, 2, -3 and graph them



(4) Graph $h(x) = |x|$.

$$h(x) = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$



Slope-Intercept Form

Consider the line

$$L: y - y_0 = m(x - x_0).$$

Performing routine algebra

$$y = mx - mx_0 + y_0$$
$$= mx + (y_0 - mx_0)$$

The value $b = y_0 - mx_0$ is the y-value of the point $(0, b)$ where L intersects the y-axis. In this way every line may be written in point-slope form

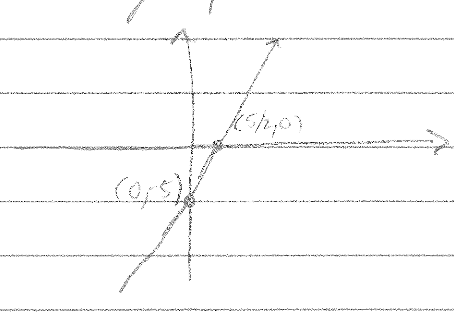
$$y = mx + b.$$

Eg: (5) What is the graph of $6x - 3y = 15$?

Put this in point-slope form:

$$y = -\tfrac{1}{3}(15 - 6x)$$
$$= 2x - 5$$

The y-intercept is $(0, -5)$, the x-intercept is $(5/2, 0)$, so the graph is



Def^n: ① We say two lines are parallel if they have the same slope.

② We say two lines are perpendicular if they have slopes m_1 and m_2 such that $m_1 m_2 = -1$.

Eg. ⑥ Find the equation of the line passing through the point $(1,3)$ parallel to the line $2x+3y=6$.

$2x+3y=6 \Rightarrow 3y = 6-2x \Rightarrow y = 2 - \frac{2}{3}x$

$\Rightarrow m = -\frac{2}{3}$.

$\Rightarrow y-3 = -\frac{2}{3}(x-1)$. ∎

⑦ Find the equation of the line passing through $(3,-2)$ and perpendicular to $y=6x+4$.

$m_1 = 6$, want $m_1 m_2 = -1 \Rightarrow m_2 = -\frac{1}{6}$

$y-(-2) = -\frac{1}{6}(x-3)$

$\Rightarrow y+2 = -\frac{1}{6}(x-3)$.

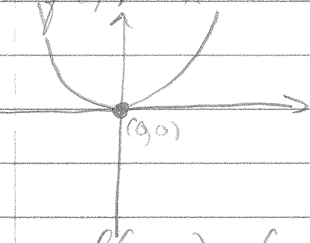
4.4 Vertical Shifts

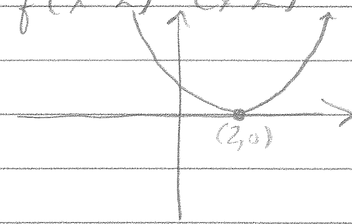
If $f(x)$ is a function, the graph of $f(x)+a$, $a>0$ is the same as the graph of $f(x)$, but shifted up by a units. If $a<0$, this is the graph of $f(x)$ shifted down by a units.

4.5 Horizontal Shifts

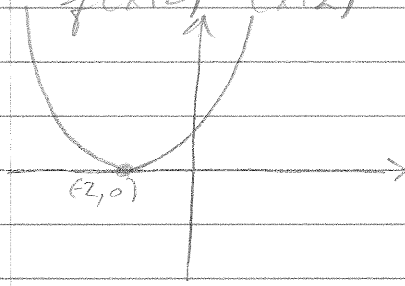
If $f(x)$ is a function, and $a \in \mathbb{R}^{\geq 0}$ then $f(x-a)$ is the same as $f(x)$ but shifted to the right by a units. The graph of $f(x+a)$ is the graph of $f(x)$ shifted left by a units.

Eg: $f(x) = x^2$ $\qquad$ $f(x-2) = (x-2)^2$





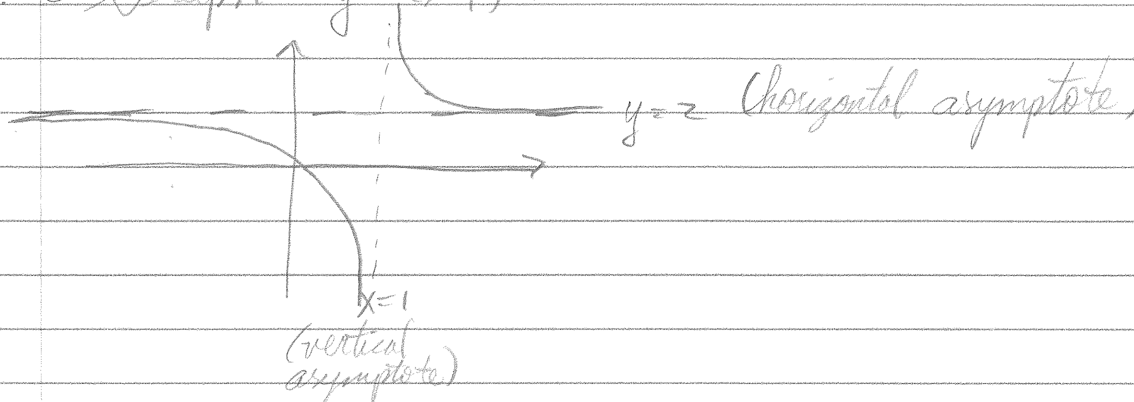
$f(x+2) = (x+2)^2$



4.6 Translations

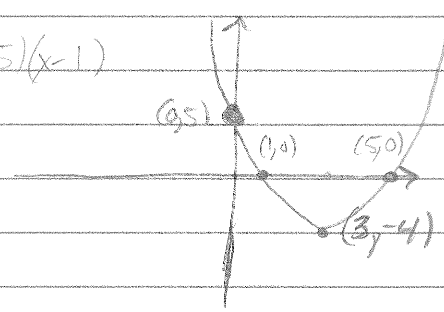
We can use these two pieces of information to graph functions from known functions.

Eg: ① Graph $y = \frac{1}{(x-1)} + 2$



② Graph $g(x) = x^2 - 6x + 5 = (x-5)(x-1)$

$g(x) = (x^2 - 2(3)x + 9) - 9 + 5$
$\quad = (x-3)^2 - 4$

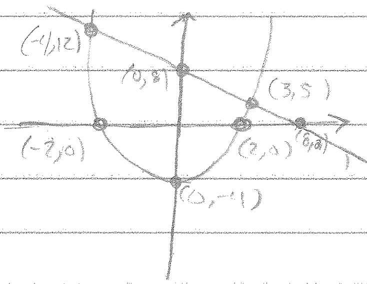


4.7 Intersections of Curves and Simultaneous Solutions

Eg.: ① Find the intersection points of the curves
$y = x^2 - 4$ and $x + y = 8$

$0 = -x + 8$
$-x = -8$

$y = -x + 8$ (slope intercept)



$x^2 - 4 = -x + 8$
$\Rightarrow x^2 + x - 12 = 0$
$\Rightarrow (x+4)(x-3) = 0$
$\Rightarrow x = 3$ or $x = -4$

Intersection points

$y = -(3) + 8 = 5 \Rightarrow (3, 5)$
$y = -(-4) + 8 = 4 + 12 = 12 \Rightarrow (-4, 12)$

② Find the point of intersection of the lines
$2x + 3y = 7$ and $-3x + y = 11$

$2x + 3y = 7$
$\Rightarrow 3y = -2x + 7$
$\Rightarrow y = -\frac{2}{3}x + \frac{7}{3}$

$-3x + y = 11$
$\Rightarrow y = 3x + 11$

$-\frac{2}{3}x + \frac{7}{3} = 3x + 11$
$\Rightarrow 3x + \frac{2}{3}x = -11 + \frac{7}{3} = \frac{-33}{3} + \frac{7}{3} = -\frac{26}{3}$
$\Rightarrow \frac{9x}{3} + \frac{2}{3}x = \frac{11}{3}x = -\frac{26}{3}$
$\Rightarrow 11x = -26$
$\Rightarrow x = -\frac{26}{11}$

Point of intersection:

$y = 3\left(\frac{-26}{11}\right) + 11 = \frac{-3(26) + 121}{11} = \frac{121 - 78}{11} = \frac{43}{11}$

$\left(\frac{-26}{11}, \frac{43}{11}\right)$