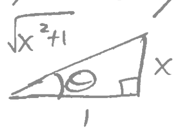
9.3

4 a) $\sin(\arctan(x))$

Let $\theta = \arctan(x)$, so $\tan(\theta) = \tan(\arctan(x)) = x$.
Construct the right triangle



which gives

$$\sin(\arctan(x)) = \sin(\theta) = \frac{x}{\sqrt{x^2+1}}.$$

This is valid for all real numbers.

b) $\tan(\operatorname{arcsec}(x))$

Let $\theta = \operatorname{arcsec}(x)$, so $\sec(\theta) = \sec(\operatorname{arcsec}(x)) = x$. Construct the right triangle

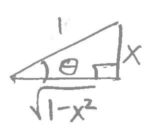
which gives

$$\tan(\operatorname{arcsec}(x)) = \tan(\theta) = \sqrt{x^2-1}.$$

Since $\operatorname{arcsec}(x)$ has domain $(-\infty,-1] \cup [1,\infty)$, the simplification is valid for this set; equivalently this is valid for $x \le -1$ or $1 \le x$.

c) $\sec(\arcsin(x))$

Let $\theta = \arcsin(x)$, so $\sin(\theta) = \sin(\arcsin(x)) = x$. Construct the triangle



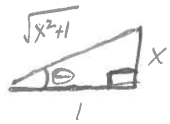
which gives

$$\sec(\arcsin(x)) = \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\sqrt{1-x^2}}.$$

This simplification is valid for $-1 < x < 1$.

d) $\sin(2\arctan(x))$

Let $\theta = \arctan(x)$, so $\tan(\theta) = \tan(\arctan(x)) = x$. Construct the triangle



which gives

$$\sin(2\arctan(x)) = \sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) = \frac{2x}{x^2+1}.$$

This simplification is valid for all real numbers.

10.4

Factor $x^3 - 7x + 6$.

Observe that

$$1^3 - 7(1) + 6 = 7 - 7 = 0$$

so by the Factor Theorem $x-1$ divides x^3-7x+6. Using polynomial division

$$\begin{array}{r|l} & \quad x^2 + x - 6 \\ \hline x-1 & x^3 \quad\; - 7x + 6 \\ & \underline{-x^3 + x^2} \\ & \quad\; x^2 - 7x \\ & \underline{\;\;- x^2 + x} \\ & \qquad -6x + 6 \\ & \qquad \underline{+6x - 6} \\ & \qquad\qquad 0 \end{array}$$

we see $x^3 - 7x + 6 = (x-1)(x^2+x-6)$. We can use the Quadratic Formula to get the roots of x^2+x-6

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)} = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2}$$

so

$$x = \frac{-1+5}{2} = \frac{4}{2} = 2 \quad \text{or} \quad x = \frac{-1-5}{2} = \frac{-6}{2} = -3.$$

Therefore by the Factor Theorem

$$x^2+x-6=(x-2)(x+3)$$

and

$$x^3-7x+6=(x-1)(x-2)(x+3).$$

10.5

6) $$\frac{x^4-36}{x+\sqrt{6}} = \frac{x^4-36}{x+\sqrt{6}}\frac{(x-\sqrt{6})}{(x-\sqrt{6})} = \frac{(x^2-6)(x^2+6)(x-\sqrt{6})}{x^2-6} = (x^2+6)(x-\sqrt{6}).$$

8) $$f(x) = \frac{1}{\sqrt{2x}}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{\sqrt{2(x+h)}} - \frac{1}{\sqrt{2x}}}{h} = \frac{1}{h}\left(\frac{\sqrt{2x}}{\sqrt{2x}\sqrt{2(x+h)}} - \frac{\sqrt{2(x+h)}}{\sqrt{2x}\sqrt{2(x+h)}}\right)$$

$$= \frac{1}{h}\left(\frac{\sqrt{2x}-\sqrt{2(x+h)}}{\sqrt{4x(x+h)}}\right)\left(\frac{\sqrt{2x}+\sqrt{2(x+h)}}{\sqrt{2x}+\sqrt{2(x+h)}}\right)$$

$$= \frac{1}{h}\left(\frac{2x-2(x+h)}{\sqrt{4x(x+h)}\left(\sqrt{2x}+\sqrt{2(x+h)}\right)}\right)$$

$$= \frac{1}{h}\left(\frac{-2h}{\sqrt{4x(x+h)}\left(\sqrt{2x}+\sqrt{2(x+h)}\right)}\right)$$

$$= \frac{-1}{\sqrt{4x(x+h)}\left(\sqrt{2x}+\sqrt{2(x+h)}\right)}$$

11.1 $f(x) = 2x^2 - 2x$

a) $f(x+h) = 2(x+h)^2 - 2(x+h)$

$= 2(x^2 + 2xh + h^2) - 2x - 2h$

$= 2x^2 + 4xh + 2h^2 - 2x - 2h$

b) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2x^2 + 4xh + 2h^2 - 2x - 2h - (2x^2 - 2x)}{h}$

$= \dfrac{2x^2 + 4xh + 2h^2 - 2x - 2h - 2x^2 + 2x}{h}$

$= \dfrac{4xh + 2h^2 - 2h}{h}$

$= 4x + 2h - 2.$