2.1

1) a) $f(x) = x^2 - 6x + 15$
$= (x^2 - 2(3)x + 9) + 6$
$= (x-3)^2 + 6$. ■

b) $h(y) = y^2 + 5y$
$= (y^2 + 2(5/2)y + (5/2)^2) - (5/2)^2$
$= (y + 5/2)^2 - \frac{25}{4}$. ■

c) $g(s) = s^2 + 2s - 8$
$= (s^2 + 2s + 1) - 9$
$= (s+1)^2 - 9$. ■

d) $k(x) = 2x^2 - 2x + 5$
$= 2(x^2 - 2(\frac{1}{2})x + (\frac{1}{2})^2) - 2(\frac{1}{2})^2 + 5$
$= 2(x - \frac{1}{2})^2 - \frac{1}{2} + \frac{10}{2}$
$= 2(x - \frac{1}{2})^2 + \frac{9}{2}$. ■

e) $f(x) = 3x^2 - 7x + 1$
$= 3(x^2 - 2(\frac{7}{2 \cdot 3})x) + 1$
$= 3(x^2 - 2(\frac{7}{6})x + (\frac{7}{6})^2) - 3(\frac{7}{6})^2 + 1$
$= 3(x - 7/6)^2 - 3(\frac{49}{36}) + 1$
$= 3(x - 7/6)^2 - \frac{49}{12} + \frac{12}{12}$
$= 3(x - 7/6)^2 - \frac{37}{12}$. ■

f) $w(x) = \pi x^2 + 2x$
$= \pi(x^2 + 2(\frac{1}{\pi})x + (\frac{1}{\pi})^2) - \pi(\frac{1}{\pi})^2$
$= \pi(x + \frac{1}{\pi})^2 - \frac{1}{\pi}$. ■

2) a) $f(x) = x^2 - 8x + 12$
$= (x^2 - 2(4)x + 16) - 4$
$= (x-4)^2 - 4$. ■

b) $h(y) = y^2 + 14y$
$= (y^2 + 2(7)y + 7^2) - 7^2$
$= (y+7)^2 - 49$. ■

c) $g(s) = s^2 + 3s - 6$

$= \left(s^2 + 2\left(\frac{3}{2}\right)s + \left(\frac{3}{2}\right)^2\right) - \left(\frac{3}{2}\right)^2 - 6$

$= (s + 3/2)^2 - \frac{9}{4} - \frac{24}{4}$

$= (s + 3/2)^2 - \frac{33}{4}$. ▨

d) $k(x) = 4x^2 - 8x + 3$

$= 4(x^2 - 2(1)x + 1) - 4(1) + 3$

$= 4(x-1)^2 - 1$. ▨

5) a) $x^2 + 3x + 2y^2 - 8y = 0$

$\Rightarrow \left(x^2 + 2\left(\frac{3}{2}\right)x + (3/2)^2\right) + 2(y^2 - 2(2)y + 2^2) = 9/4 + 2(2)^2 = \frac{9}{4} + \frac{32}{4} = \frac{41}{4}$

$\Rightarrow (x + 3/2)^2 + 2(y-2)^2 = 41/4$. ▨

b) $3x^2 + 6x - 2y^2 - 8y = -11$

$\Rightarrow 3(x^2 + 2x + 1) - 2(y^2 + 2(2)y + 2^2) = -11 + 3 - 8 = -16$

$\Rightarrow 3(x+1)^2 - 2(y+2)^2 = -16$. ▨

c) $-x^2 + 4x + y^2 - 16y = 40$

$\Rightarrow -(x^2 - 2(2)x + 4) + (y^2 - 2(8)y + 8^2) = 40 - 4 + 64 = 100$

$\Rightarrow -(x-2)^2 + (y-8)^2 = 100$. ▨

d) $-9x^2 + 36x - 4y^2 - 8y = 0$

$\Rightarrow -9(x^2 - 2(2)x + 4) - 4(y^2 + 2(1)y + 1) = -36 - 4 = -40$

$\Rightarrow -9(x-2)^2 + 4(y+1)^2 = 40$. ▨

e) $x^2 + y^2 - 6x + 10y + 34 = 0$

$\Rightarrow (x^2 - 2(3)x + 9) + (y^2 + 2(5)y + 25) - 9 - 25 + 34 = 0$

$\Rightarrow (x-3)^2 + (y+5)^2 = 0$. ▨

Rmk: This is degenerate because it has radius 0.
It consists only of the point $(3, -5)$.

7) a) $x^2+y^2-4x-2y=11$

$\Rightarrow (x^2-2(2)x+4)+(y^2-2(1)y+1)=11+4+1$

$\Rightarrow (x-2)^2+(y-1)^2=16.$

Center: $(2,1)$,

Radius: $\sqrt{16}=4.$ ▨

b) $x^2+y^2-6x+4y-\pi^2+13=0$

$\Rightarrow (x^2-2(3)x+9)+(y^2+2(2)y+4)-\pi^2+13-9-4=0$

$\Rightarrow (x-3)^2+(y+2)^2=\pi^2$

Center: $(3,-2)$

Radius: $\pi.$ ▨

c) $2x^2+2y^2+4x+8y-20=0$

$\Rightarrow x^2+y^2+2x+4y=10$

$\Rightarrow (x^2+2x+1)+(y^2+2(2)y+4)=10+1+4=15$

$\Rightarrow (x+1)^2+(y+2)^2=15.$

Center: $(-1,-2)$

Radius: $\sqrt{15}.$ ▨

8) a) $x^2+y^2-6x-8y=0$

$\Rightarrow (x^2-2(3)x+9)+(y^2-2(4)y+16)=9+16=25$

$\Rightarrow (x-3)^2+(y-4)^2=25$

Center: $(3,4)$

Radius: $5.$ ▨

b) $x^2+y^2-10x+12y+12=0$

$\Rightarrow (x^2-2(5)x+25)+(y^2+2(6)y+36)=-12+25+36=49$

$\Rightarrow (x-5)^2+(x+6)^2=49$

Center: $(5,-6)$

Radius: $7.$ ▨