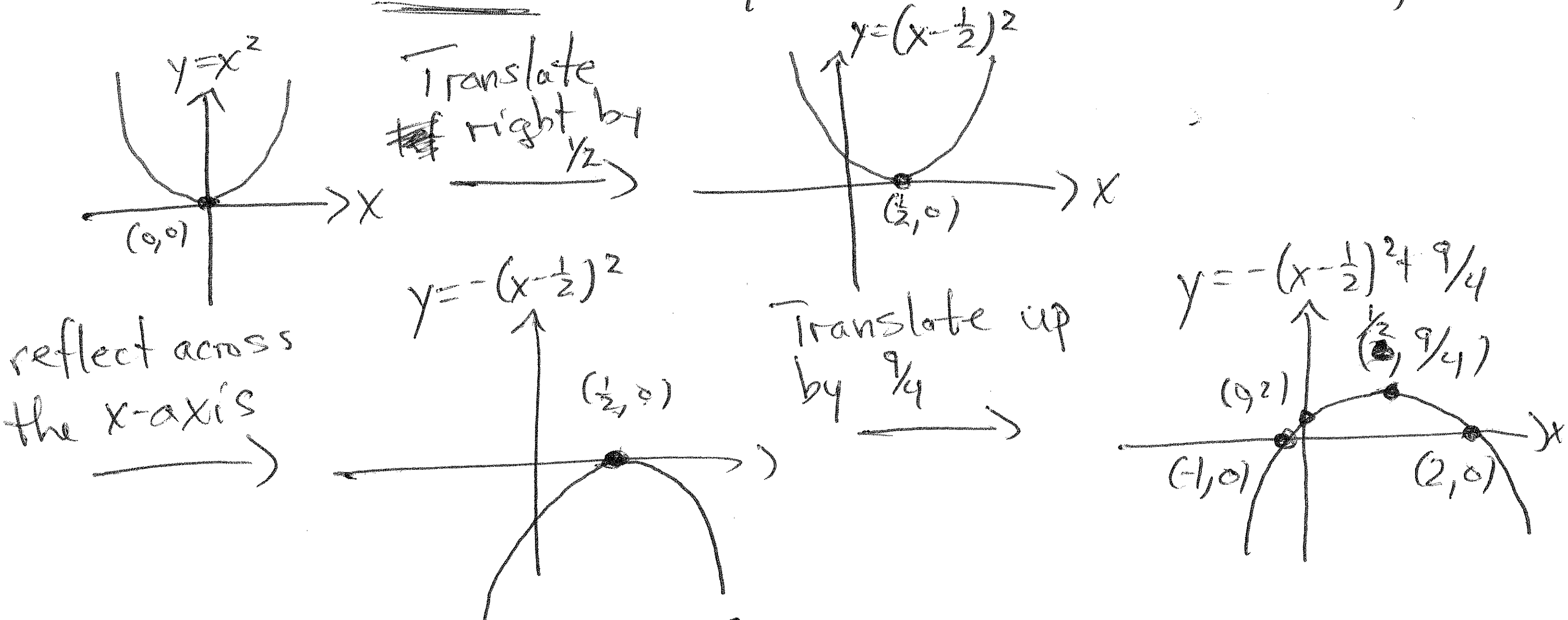
Eg: $f(x) = -x^2 + x + 2$

Graph $f(x)$ by placing it in standard form and using translations.

$$\boxed{x^2 - 2ax + a^2 = (x-a)^2}$$

$$\begin{aligned}
f(x) &= -x^2 + x + 2 \quad \text{(general Form)} \\
&= -(x^2 - x) + 2 \\
&= -(x^2 - 2(\tfrac{1}{2})x) + 2 \\
&= -(\underbrace{x^2 - 2(\tfrac{1}{2})x + (\tfrac{1}{2})^2} - (\tfrac{1}{2})^2) + 2 \\
&= -((x - \tfrac{1}{2})^2 - \tfrac{1}{4}) + 2 \\
&= -(x - \tfrac{1}{2})^2 + \tfrac{1}{4} + \tfrac{8}{4} \\
&= -\underline{(x - \tfrac{1}{2})^2} + \tfrac{9}{4} \quad \text{(Standard Form)}
\end{aligned}$$



$f(x) = 0 = -x^2 + x + 2$ $\qquad x = -1$ or $x = 2$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)(2)}}{2(-1)} = \frac{-1 \pm \sqrt{1+8}}{-2} = \frac{-1 \pm \sqrt{9}}{-2} = \frac{-1 \pm 3}{-2}$$

Rmk: $-(x+1)(x-2) = -(x^2-2x+x-2)$

$$= -(x^2-x-2)$$

$$= -x^2+x+2$$

$$= f(x).$$

Graph $f(x)$ using the vertex formula

Recall: The x-coordinate of ~~f(x)~~ the vertex of $f(x)$ is $-b/2a$

$$f(x) = -x^2+x+2$$

$\Rightarrow$ vertex has x-coordinate

$$\frac{-(1)}{2(-1)} = \frac{-1}{-2} = \frac{1}{2}.$$

The y-coordinate of the vertex is

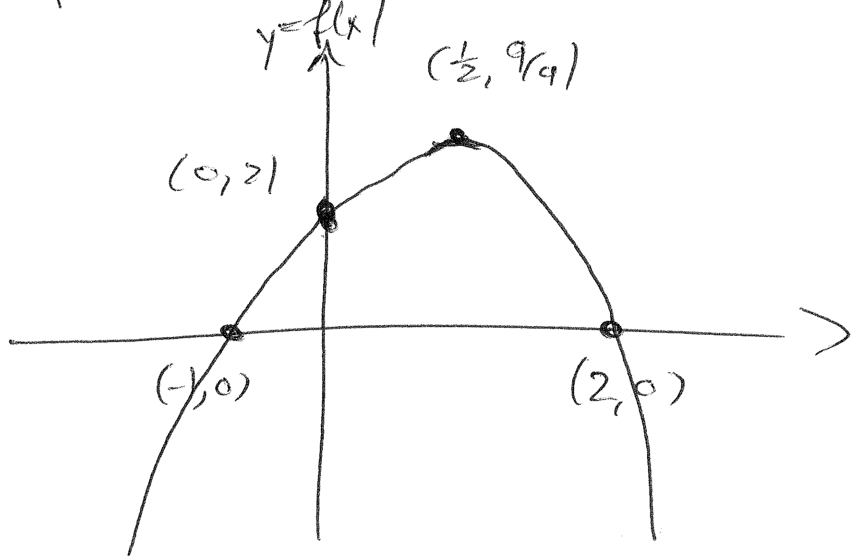
$$f(\tfrac{1}{2}) = -(\tfrac{1}{2})^2 + \tfrac{1}{2} + 2$$

$$= -\frac{1}{4} + \frac{2}{4} + \frac{8}{4}$$

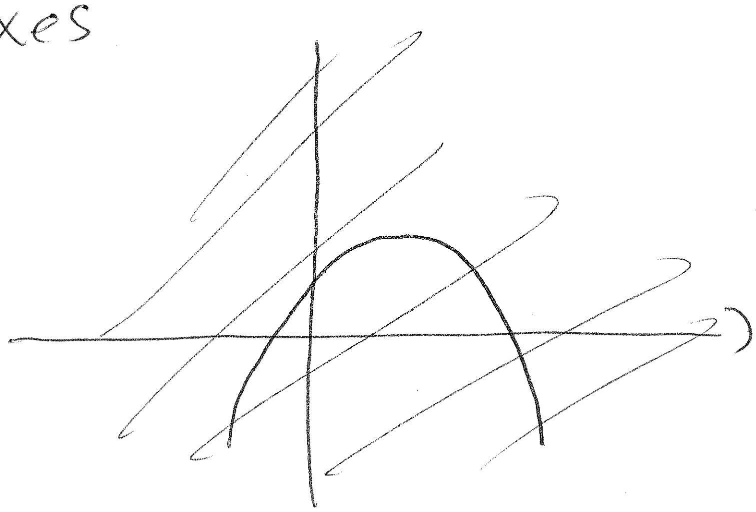
$$= \frac{9}{4}.$$

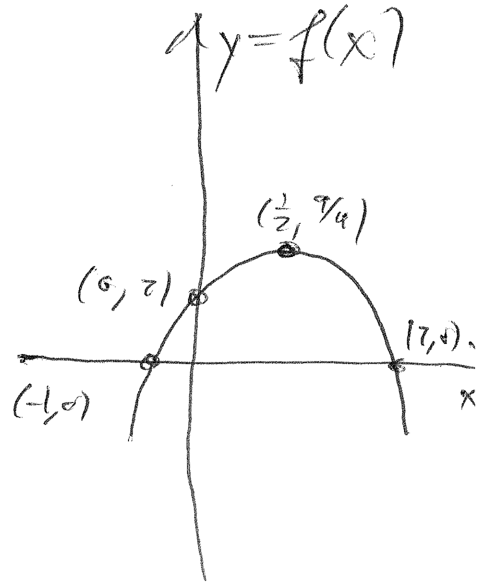
The roots are ~~x~~ $(-1,0)$ and $(2,0)$ and the y-intercept is $(0,2)$.

Since the value of a is negative, the parabola opens down

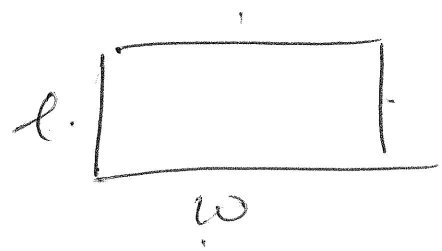


Draw the upside parabola, then fill in the axes





Eg: Have 140 ft of fencing to fence a rectangular garden.



$$P(l, w) = 2l + 2w = 140$$

$$A(l, w) = l \cdot w$$

Solve $2l + 2w = 140$ for l

$$\Rightarrow 2l = 140 - 2w$$

$$\Rightarrow l = 70 - w$$

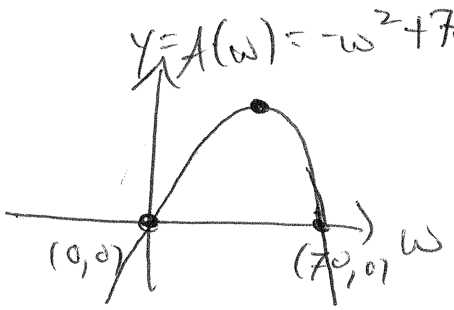
$$A(l, w) = A(70 - w, w)$$

$$= (70 - w)\, w$$

$$= 70w - w^2.$$

$$= -w^2 + 70w$$ (~~Star~~ General Form)

Downward facing parabola. The vertex is the maximum of this function, the graph looks like



and the w-coordinate of the vertex is

$$\frac{-70}{2(-1)} = \frac{-70}{-2} = 35.$$

So $l = 70 - w = 70 - 35$ gives us that the maximal area with 140 feet of fencing is a 35×35 square garden (Area 1225)

E.g.: $f(x) = x^2 + 4x$

Find the min/~~max~~.

$a = 1 > 0$, parabola opens up, min occurs at the vertex with x-coord

$$x = \frac{-4}{2(1)} = -2$$

and y-coord

$$f(-2) = (-2)^2 + 4(-2)$$
$$= 4 - 8$$
$$= -4.$$

$(-2, -4)$

The minimum value of $f(x)$ is the y-coordinate of the vertex, -4.