E.g:

Recall: If $a, b \neq 1$ are positive numbers,

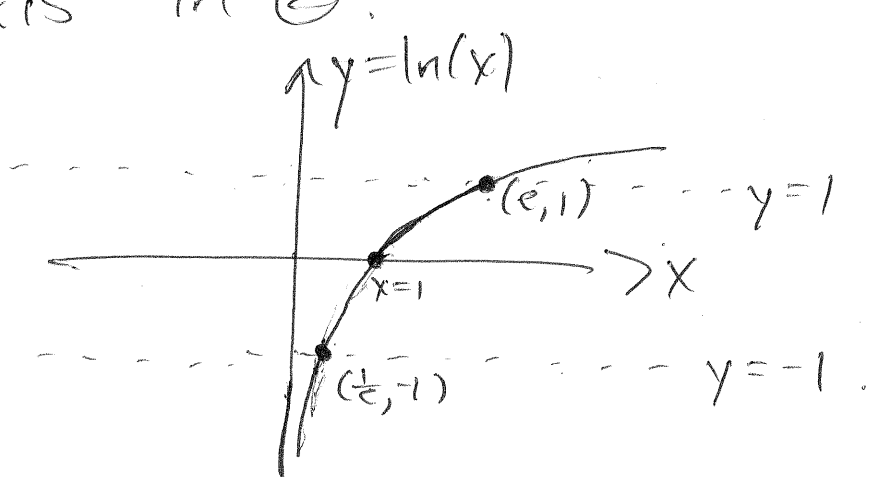
$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}.$$

In particular,

$$\log_b(x) = \frac{\ln(x)}{\ln(b)} = \frac{1}{\ln(b)} \cdot \ln(x)$$

$$\ln(b) = \begin{cases} \text{positive if } b > 1, & \text{①} \\ \text{negative if } 0 < b < 1. & \text{②} \end{cases}$$

The graph of $\log_b(x)$ is a stretching/shrinking of $\ln(x)$ in ① and a stretching/shrinking of $\ln(x)$ with a reflection across the x-axis in ②.



Def^n: A quadratic function is a function of the form

$$f(x) = ax^2 + bx + c$$

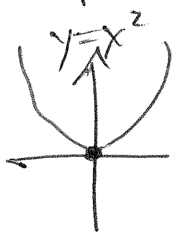
a, b, c are all real numbers, $a \neq 0$.
This is called general form.

<u>Standard Form</u>

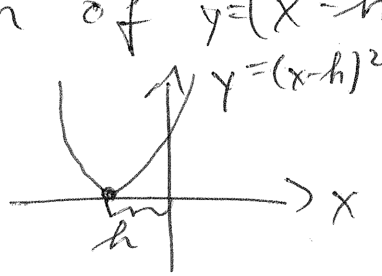
$f(x) = ax^2 + bx + c$, there exists real numbers h and k such that

$$f(x) = a(x-h)^2 + k.$$

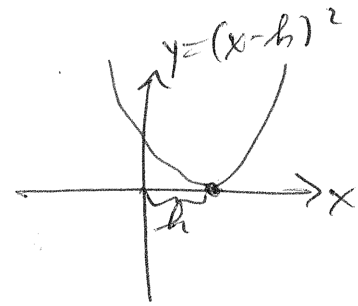
Start from the graph of $\cancel{f(x)}$ $y = x^2$



then translate horizontally by h to get the graph of $y = (x-h)^2$



or



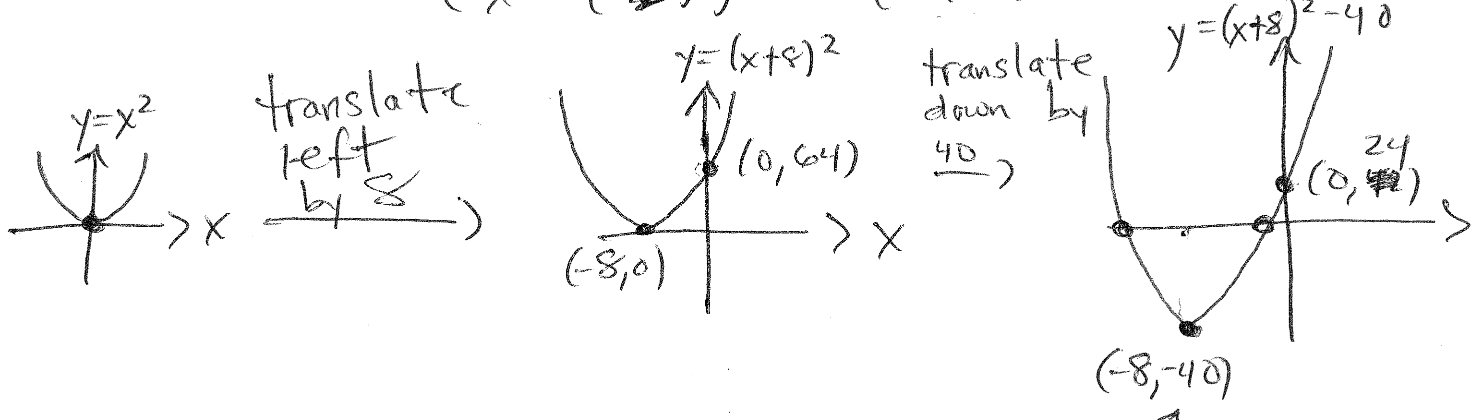
③ Stretch or shrink the graph of $y=(x-h)^2$ by $|a|$ to get the graph of $y=|a|(x-h)^2$.

④ If $a<0$, reflect across the x-axis. This gives the graph of $y=a(x-h)^2$.

⑤ Translate vertically by k (up if $k>0$, down if k is negative) to get the graph of

$$y=f(x)=a(x-h)^2+k.$$

E.g.:
$$f(x)=(x+8)^2-40 \qquad h=-8$$
$$=(x-(-8))^2+(-40) \qquad k=-40.$$



$f(x)=(x+8)^2-40=x^2+16x+24$ vertex

Rmk: The vertex of $f(x)=a(x-h)^2+k$ is (h,k).

How to put a quadratic function into Standard form

$$f(x) = ax^2 + bx + c$$ factor a out of the first 2 terms

$$= a\left(x^2 + 2\left(\frac{b}{2a}\right)x\right) + c$$ complete the square inside ( )

$$= a\left(x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$ factor

$$= a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$ distribute the a

$$= a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$$

$$= a\left(x - \underbrace{\left(\frac{-b}{2a}\right)}_{h}\right)^2 + \underbrace{\left(c - \frac{b^2}{4a}\right)}_{k}$$

<u>Corollary</u>: The x-coordinate of the vertex of $f(x) = ax^2 + bx + c$ is $-b/2a$.

<u>Eg</u>: $f(x) = x^2 + 3x + 1 = \left(x - \left(\frac{-3}{2}\right)\right)^2 + \left(\frac{-5}{4}\right)$.

$$x = \frac{-3}{2(1)} = -\frac{3}{2}$$

$$f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) + 1$$
$$= \frac{9}{4} - \frac{9}{2} + 1 = \frac{9}{4} - \frac{18}{4} + \frac{4}{4} = \frac{-5}{4}.$$

Eg: $f(x) = 2x^2 - 12x + 23$

$$h = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$$

$$k = f(3) = 2(3)^2 - 12(3) + 23$$
$$= 2(9) - 36 + 23$$
$$= 18 + 23 - 36$$
$$= 5$$

$$\begin{array}{r} 23 \\ 18 \\ \hline 41 \\ -36 \\ \hline 5 \end{array}$$

Vertex: $(3, 5)$

y-intercept: $(0, f(0)) = (0, 23)$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{array}{r} 2 \\ 23 \\ 8 \\ \hline 184 \end{array}$$

$$b^2 - 4ac = (-12)^2 - 4(2)(23)$$
$$= 144 - 184 < 0$$