Eg.: Logarithmic Equations

$$\log(x+1) - \log(x) = 2$$

$$\Rightarrow \log\left(\frac{x+1}{x}\right) = 2$$

$$\Rightarrow 10^{\log\left(\frac{x+1}{x}\right)} = 10^2$$

$$\Rightarrow \frac{x+1}{x} = 10^2$$

$$\Rightarrow x+1 = 10^2 \cdot x$$

$$\Rightarrow 1 = 10^2 x - x$$

$$\Rightarrow 1 = x(10^2 - 1)$$

$$\Rightarrow 1 = x \cdot 99$$

$$\Rightarrow x = 1/99.$$

5: Quadratic Functions & Models

5.1 Working w/functions: shifting & stretching

We would like to graph something like

$$f(x) = x^2 + 5x + 6.$$

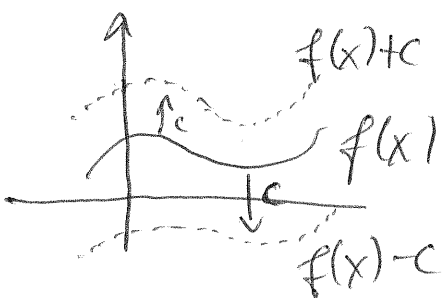
We'll look at basic ~~translations~~ transformations of graphs to answer this question.

Vertical Shift/Translation

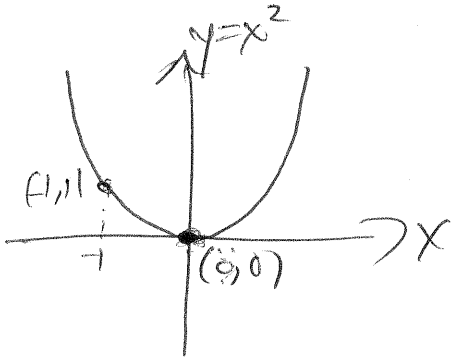
Suppose $c > 0$ and the graph of a function $f(x)$ is given, then the graph of

$y = f(x) + c$ - vertical translation of the graph of $f(x)$ up by c units

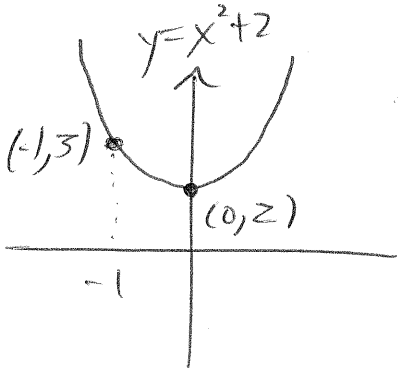
$y = f(x) - c$ - vertical ~~#~~ translation of the graph of $f(x)$ down by c units.



E.g.: Graph $x^2 + 2$



translate up by 2.



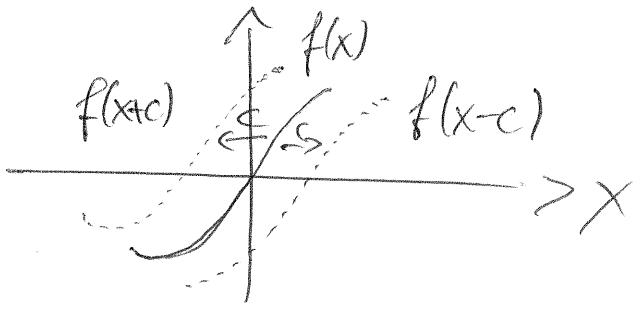
Fixes the x-coordinate and increases the y-coordinate by 2.

Horizontal Shift

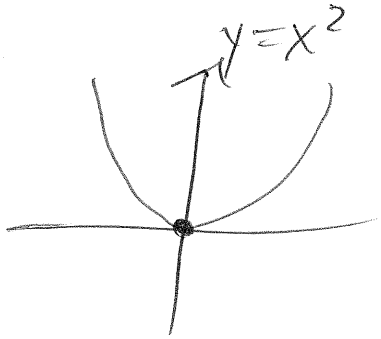
Suppose $c>0$, given the graph of $f(x)$.

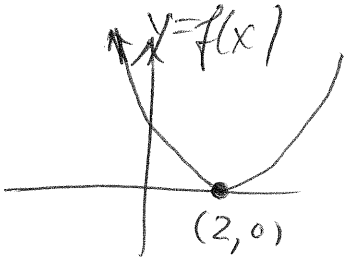
The graph of $y=f(x+c)$ is the graph of $f(x)$ shifted c units to the left.

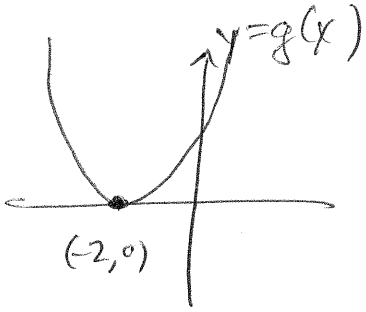
The graph of $y=f(x-c)$ is the graph of $f(x)$ shifted c units to the right.



Eg: $f(x) = (x-2)^2$ $\quad g(x) = (x+2)^2$





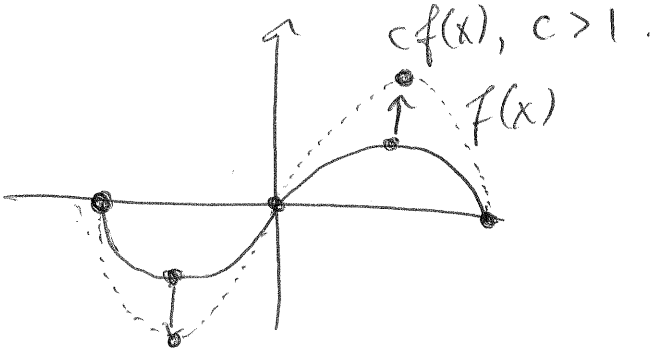


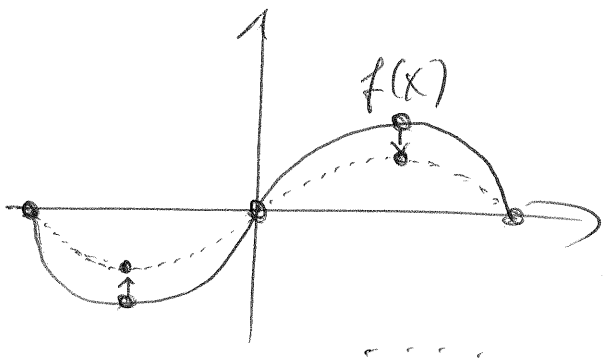
Vertical Stretching & Shrinking

$y = cf(x)$, $c > 0$

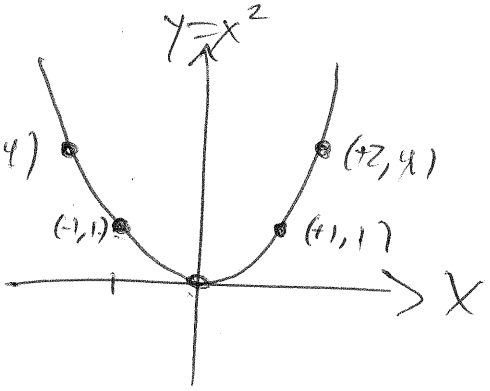
If $c > 1$, stretch $f(x)$ vertically by a factor of c.

If $0 < c < 1$, shrink $f(x)$ vertically by a factor of c.

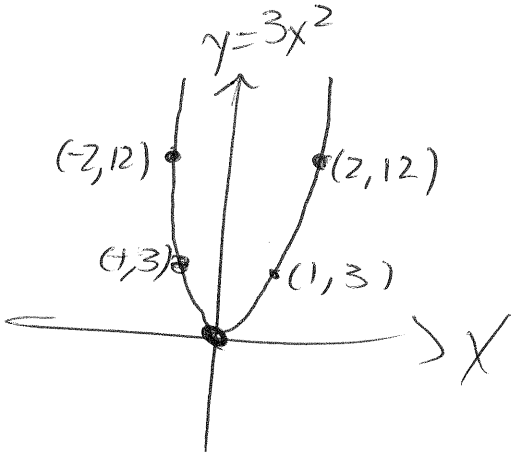




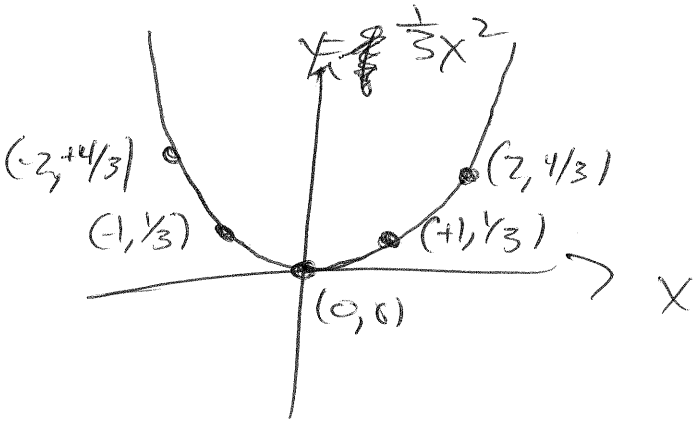
Eg: $f(x) = 3x^2$



stretch by 3



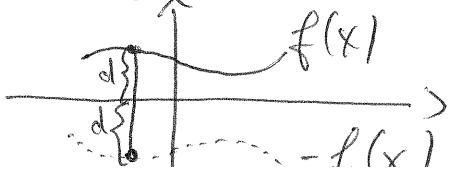
shrink by 1/3

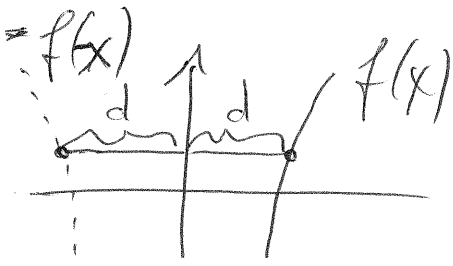


Reflections

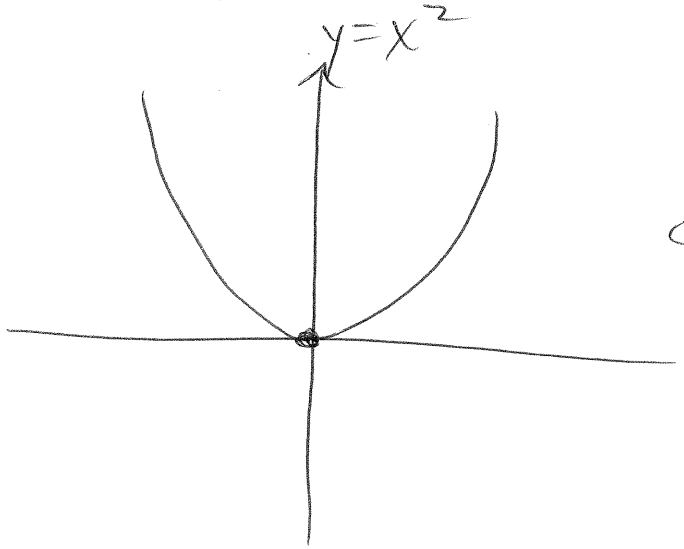
To graph $y = -f(x)$, reflect across the x-axis

To graph $y = f(-x)$, reflect across the y-axis.

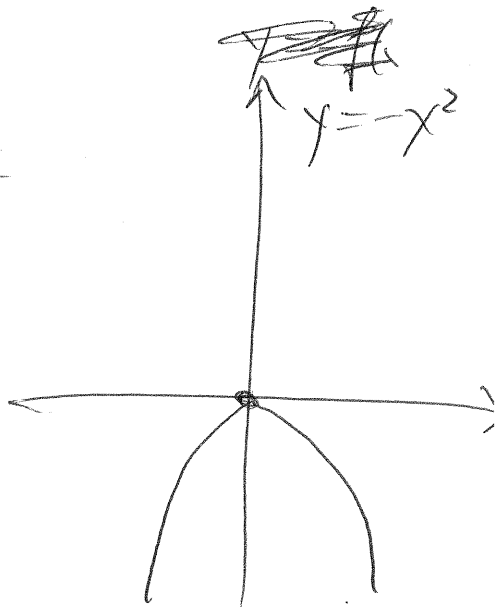




$f(x) = -x^2$



reflect across the x-axis



Back to $f(x) = x^2 + 5x + 6$.

Use completing the square to do the following:

$(x+a)^2 = x^2 + 2ax + a^2$

$f(x) = x^2 + 5x + 6$

$= x^2 + 2\left(\frac{5}{2}\right)x + 6$

$= x^2 + 2\left(\frac{5}{2}\right)x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6$

$= (x + 5/2)^2 - \frac{25}{4} + \frac{24}{4}$

$= (x + 5/2)^2 - \frac{1}{4}$.