The exponential growth or decay model

$$f(x) = Ca^x \quad (a - \text{growth factor per time period})$$

is equivalent to

$$f(x) = Ca^x = C\left(e^{\ln(a)}\right)^x = Ce^{\ln(a)x}.$$

where the instantaneous growth/decay rate is

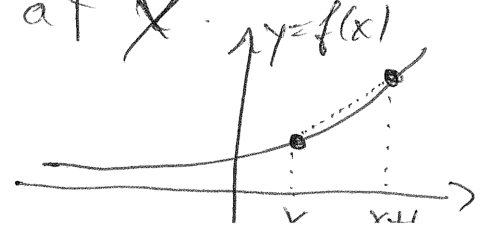
$$r = \ln(a).$$

<u>Fact:</u>
The instantaneous rate of change of a function $f(x) = Ce^{rx}$ is $r(Ce^{rx})$.

<u>Recall:</u> For $f(x) = Ca^x$, the growth rate was defined to be

$$\frac{\left(\frac{f(x+1) - f(x)}{(x+1) - x}\right)}{f(x)} = \frac{f(x+1) - f(x)}{f(x)}$$

Average rate of change as a percentage of the value of f at x.



For Ce^{rx}, the instantaneous ~~rate~~ growth/decay rate is the ~~r~~ value

$$r = \frac{r \cdot Ce^{rx}}{Ce^{rx}} \leftarrow \text{instantaneous rate of change}$$

Eg From ~~Wednesday~~ Monday

$f(x) = 350(1.4)^x$ growth rate: $1.4 - 1 = .4$

$f(x) = 350\, e^{\ln(1.4)x}$ instaneous growth rate $\ln(1.4) \approx 0.34$.

Eg: 22 micrograms of a radioactive substance. Amount decreases by 10% / day.

Daily ~~growth~~ decay rate: $r = 0.1$

Daily ~~growth~~ decay factor: $a = 1 + r = \cancel{}\ 1 + (-0.1)$
$= 0.9$.

$$A(t) = 22(0.9)^t \text{ (micrograms)}$$
$$= 22(e^{\ln(0.9)})^t$$
$$= 22\, e^{\ln(0.9)t}$$

Instantaneous growth rate $r = \ln(0.9) \approx -0.105$.

4.5: Exponential Equations: Getting Information From a Model. ③

Crucial: $e^{\ln(x)} = x$ $\qquad a^{\log_a(x)} = x$

$\ln(e^x) = x$ $\qquad \log_a(a^x) = x.$

E.g.: $2^x = 9.$

$\log(2^x) = \log(9)$

$\Rightarrow x\log(2) = \log(9)$

$\Rightarrow \quad x = \dfrac{\log(9)}{\log(2)} \approx 3.17.$

$\log_2(2^x) = \log_2(9)$

$\Rightarrow x = \log_2(9).$

$\Rightarrow x = \dfrac{\log(9)}{\log(2)}$

$= \dfrac{\ln(9)}{\ln(2)}$

E.g.: $5\cdot 2^x = 36$

$\log(5\cdot 2^x) = \log(36)$

$\Rightarrow \log(5) + \log(2^x) = \log(5) + \underline{x}\log(2) = \log(36)$

$\Rightarrow x\log(2) = \log(36) - \log(5) = \log(36/5)$

$\Rightarrow \quad x = \dfrac{\log(36/5)}{\log(2)} \approx 2.848$