$$\frac{f(x+1)-f(x)}{f(x)} = \frac{10\cdot 3^{x+1} - 10\cdot 3^x}{10\cdot 3^x}$$

$$= \frac{(\cancel{10\cdot 3^x})3}{\cancel{10\cdot 3^x}} - \frac{\cancel{10\cdot 3^x}}{\cancel{10\cdot 3^x}}\ 1$$

$$= 3 - 1 = 2$$

$\underline{x=0}$

$10\cdot 3^0 = 10$

$\underline{x=1}$

$10\cdot 3^1 = 30$

$$\frac{30-10}{10} = \frac{20}{10}$$

$\underline{x=2}$

$10\cdot 3^2 = 10\cdot 9 = 90$

$$\frac{90-30}{30} = \frac{60}{30} = 2.$$

<u>Avg</u> <u>Rate</u> <u>of</u> <u>Change</u>

From 0 to 1

$30-10=20$

From 1 to 2

$90-30=60$

From 2 to 3

$270-90=180$

From 3 to 4

$810-270=540$

Percentage rate of change is 200%.

Average rate of change over intervals of length 1 is <u>not</u> constant.

$$\frac{f(x+1)-f(x)}{(x+1)-x} = f(x+1)-f(x)$$

Eg.: Data Set Given

x	y
0	10,000
1	7,000
2	4900
3	3430
4	2401

Find Avg R.o.C. and Percentage R.o.C. for consecutive outputs

Avg. R.o.C.

$7{,}000 - 10{,}000 = -3{,}000$

$4900 - 7000 = -2{,}100$

$3430 - 4900 = -1470$

$2401 - 3430 = -1029$

Linear model will not accurately represent this data.

Percentage R.o.C.

$\frac{7{,}000 - 10{,}000}{10{,}000} = \frac{-3{,}000}{10{,}000} = -30\%$

$\frac{2100}{7000} = -30\%$

$\frac{-1470}{4900} = -30\%$

$\frac{1029}{3430} = -30\%$

This suggest that an exponential model is appropriate here.

We know the $r = -30/100 = -.3$, so $a = 1 + r = 1 - .3 = .7$

$f(x) = 10{,}000(.7)^x$

$f(x) = mx+b$

$f(x+1) = m(x+1) + b$
$= (mx+b) + m$
$= f(x) + m$

$f(x+1) - f(x) = (f(x) + m) - f(x)) = m$

$g(x) = Ca^x$

$g(x+1) - g(x) = Ca^{x+1} - Ca^x$
$= Ca^x \cdot a - Ca^x$
$= Ca^x(a-1)$
$= r\, Ca^x$

E.g.: Population 10,000
Two models of growth over 5 years

A: population increases by 500 people each year
B: population increases by 5% per year.

a) Use ~~model~~ A's estimate to model $P_A(t)$ for the population in t years.

$P_A(t) = 10000 + 500t$

Average rate of change of the population increase/year is 500

c) Use estimate B to find a model $P_B(t)$ for the population t years from now. (4)

$$r = 5/100 = 0.05$$

$$a = r + 1 = 1 + 0.05 = 1.05.$$

$$P_B(t) = 10{,}000(1.05)^{t}$$

t	$P_A(t)$
0	10000
1	10,500
2	11,000
3	11,500
4	12,000
5	12,500

t	$P_B(t)$
0	10000
1	10500
2	11,025
3	11,576
4	12,155
5	12,762

Logistic Growth

A logistic growth model is a function of the form

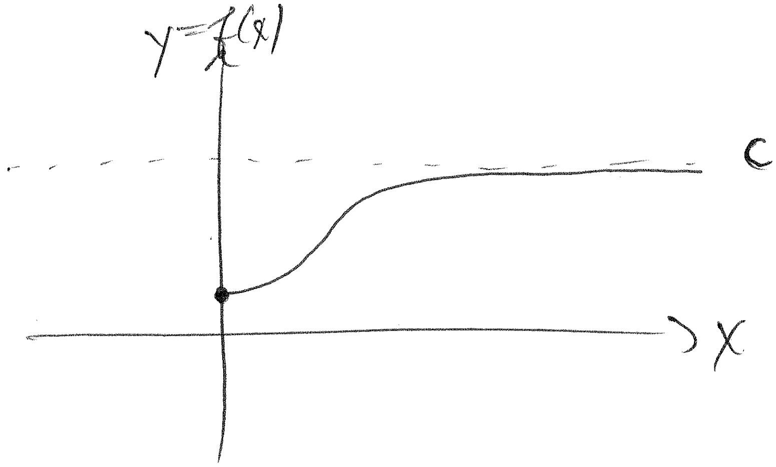
$$f(x) = \frac{c}{1 + ba^{-x}}, \quad a > 1, b > 0$$

and models growth under limited resources.

x – # of time periods,

c – carrying capacity

Graph of such a function looks like



$$f(x) = \frac{c}{1 + ba^{-x}} \qquad a > 1,\ b > 0$$

Consider $a = 10,\ b = 1.$ $\quad x > 0$

$$a^{-x} = 10^{-x} = \frac{1}{10^x}$$

$$f(x) = \frac{2}{1 + 10^{-x}}$$

$\underline{x = 0}$ $\quad \frac{1}{10^0} = 1 \qquad \frac{2}{1+1} = \frac{2}{2} = 1$

$\underline{x = 1}$ $\quad \frac{1}{10} = .1 \qquad \frac{2}{1.1} = 1.8181\ldots$

$\underline{x = 2}$ $\quad \frac{1}{10^2} = .01 \qquad \frac{2}{1.01} = 1.98019802\ldots$

$\underline{x = 3}$ $\quad \frac{1}{10^3} = .001 \qquad \frac{2}{1.001} = 1.998002\ldots$

$$\frac{2}{1.0001} = 1.99980002\ldots$$