Expanding & Combining Logarithmic Expressions ①

Eg: a) $\log_2(6x) = \log_2(6) + \log_2(x)$

Say you want to solve $\log_2(6x) = 5$ for x.

$$\log_2(6x) = \underline{\log_2(6)} + \log_2(x) = 5$$

$$\Rightarrow \quad \log_2(x) = 5 - \log_2(6)$$

$$f(\underline{x}) = 2^x$$

$$\Rightarrow f(\log_2(x)) = f(5 - \log_2(6))$$

$$\Rightarrow 2^{\log_2(x)} = x = 2^{(5-\log_2(6))}$$

$$\Rightarrow x = \frac{2^5}{2^{\log_2(6)}} = \frac{2^5}{6} = \frac{32}{6}$$

b) $\log_4\left(\frac{z^2}{y}\right) = \log_4(z^2) - \log_4(y)$

$$= 2\log_4(z) - \log_4(y)$$

c) $\log_5(x^3y^6) = \log_5(x^3) + \log_5(y^6)$

$$= 3\log_5(x) + 6\log_5(y)$$

$\log_5((xy^2)^3) = 3\log_5(xy^2) = 3(\log_5(x) + \log_5(y^2))$

$$= 3(\log_5(x) + 2\log_5(y))$$

$= 3\log_5(x) + 6\log_5(y).$

d) $\log\left(\frac{ab}{\sqrt[3]{c}}\right) = \log(ab) - \log(\sqrt[3]{c})$

$= \log(ab) - \log(c^{1/3})$

$= \log(a) + \log(b) - \frac{1}{3}\log(c).$

Eg.: Combining

a) $3\log(x) + 2\log(x-5) = \log(x^3) + \log((x-5)^2)$

$= \log(x^3(x-5)^2)$

b) $3\log(s) - \frac{1}{2}\log(t+1) = \log(s^3) - \log(\sqrt{t+1})$

$= \log\left(\frac{s^3}{\sqrt{t+1}}\right).$

Change of base formula

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Eg.: $\log_2(15) = \frac{\log(15)}{\log(2)}.$