3.4: Graphs of Exponential Functions

Defn: An exponential function with base a is a function of the form

$$f(x) = a^x$$

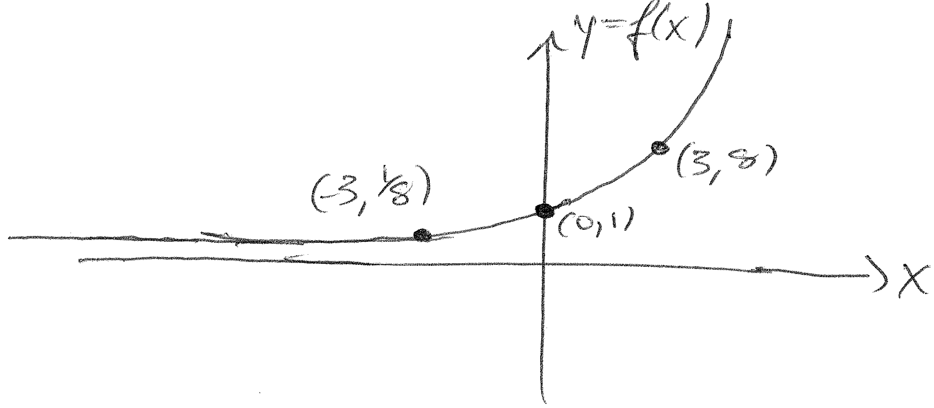
where $a > 0$, $a \neq 1$. The domain of f is the set of all real numbers.

Recall when $x > 0$

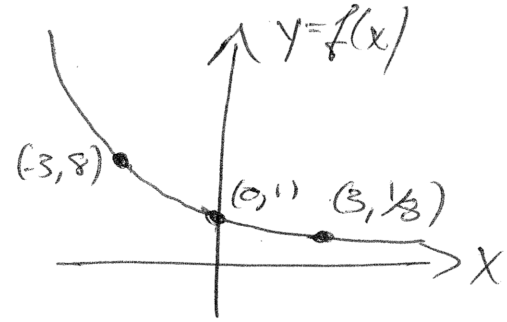
$$f(-x) = a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x.$$

Eg: $f(x) = 2^x$

Note that $f(0) = 2^0 = 1$.

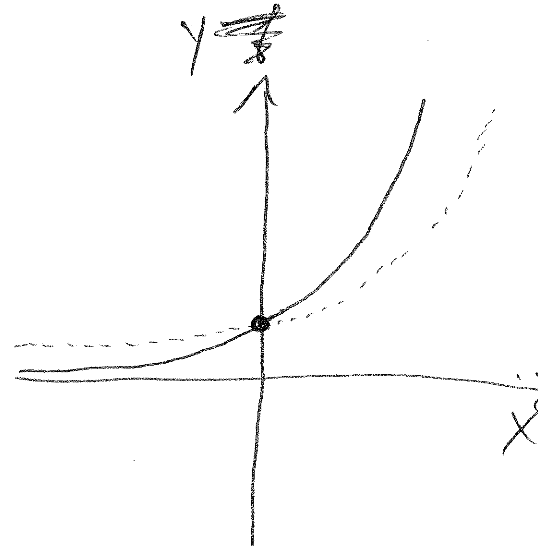


Eg: $f(x) = \left(\frac{1}{2}\right)^x$
$= 2^{-x}$



$f(x) = 3^x \qquad g(x) = 2^x$

x	$f(x)$	$g(x)$
-3	$1/27$	$1/8$
-2	$1/9$	$1/4$
-1	$1/3$	$1/2$
0	1	1
1	3	2
2	9	4
3	27	8



— $f(x)$
.... $g(x)$

Informally, an asymptote of a function is a line to which the graph of the function gets closer and closer as one travels along the line.

If $a > 1$, then $f(x) = a^x$ has a horizontal asymptote, the x-axis, as $f(x)$ tends toward 0 as x becomes large and negative.

If $a < 1$, then $f(x) = a^x$ also has a horizontal asymptote at $y = 0$ as $f(x)$ tends toward 0 as x becomes large and positive. See p. 288 of the book for a synopsis.

Eg.: In 2000 pop was 6.1 billion, growth is exponential. $r = 0.014$

<u>Claim:</u> Reducing r to 0.01 would make a significant difference in just a few decades.

a) Find a model for both.

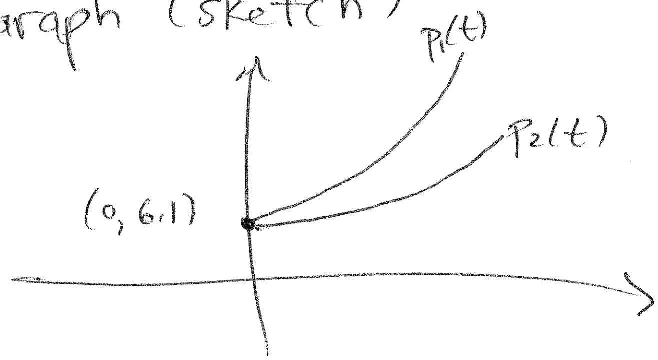
$r_1 = 0.014$ $\quad$ $r_2 = 0.01$

$a_1 = 1.014$ $\quad$ $a_2 = 1.01$

$P_1(t) = 6.1(1.014)^t$ $\quad$ $P_2(t) = 6.1(1.01)^t$

both are in billions.

Graph (sketch)

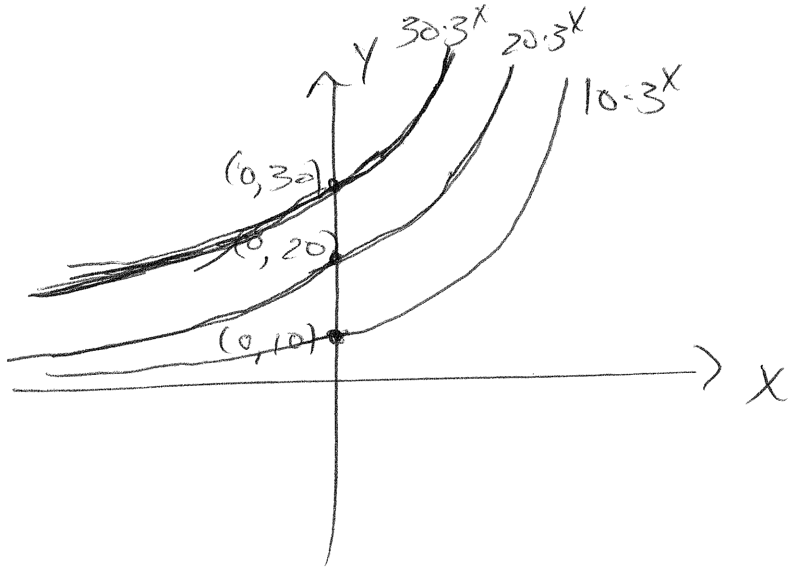


After 5 decades,

$P_1(50) = 6.1(1.014)^{50} \approx 12.2$ billion

$P_2(50) = 6.1(1.01)^{50} \approx 10$ billion.

Eg: $f(x) = C \cdot 3^x$, $C = 10, 20, 30$



4: Logarithmic Functions & Exponential Models

4.1: Logarithmic Functions

Def^n: The logarithm base 10 of x is defined by

$$\log_{10}(x) = y \text{ if and only if } 10^y = x.$$

Eg: $\log_{10}(100) = 2$ because $10^2 = 100$.

Eg: $\log_{10}\left(\frac{1}{1000}\right) = y \iff 10^y = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

$\iff y = -3.$

E.g.: $\log_{10}(735) = 2.866287339.$

Rmk: $\log_{10}(x)$ is commonly called $\log(x)$.

Def^n: If a is a positive number, then the logarithm base a of x is defined by

$$\log_a(x) = y \iff a^y = x.$$

E.g.: $\log_2(8) = \log_2(2^3) = y \iff 2^y = 2^3 = 8$

$\iff y = 3.$

E.g.: $\log_3(27) = 3$ because $3^3 = 27.$

E.g.: $\log_5(125) = 3$ because $5^3 = 5^2 \cdot 5$
$= 25 \cdot 5$
$= 125$

$\log_5(625) = 4.$

$$\begin{array}{r} 125 \\ \times\ 5 \\ \hline 625 \end{array} = 5^4$$

E.g.: $\log_4(64) = 3.$ because $4^3 = 64.$

$2^6 = 64$
$= (2^2)^3$
$= 4^3$

1. $\log_a(1) = 0$ because $a^0 = 1$
2. $\log_a(a) = 1$ because $a^1 = a$
3. $\log_a(a^x) = x$ because $a^x = a^x$.
4. $a^{\log_a(x)} = x$

by definition $\log_a(x) = y \iff a^y = x$